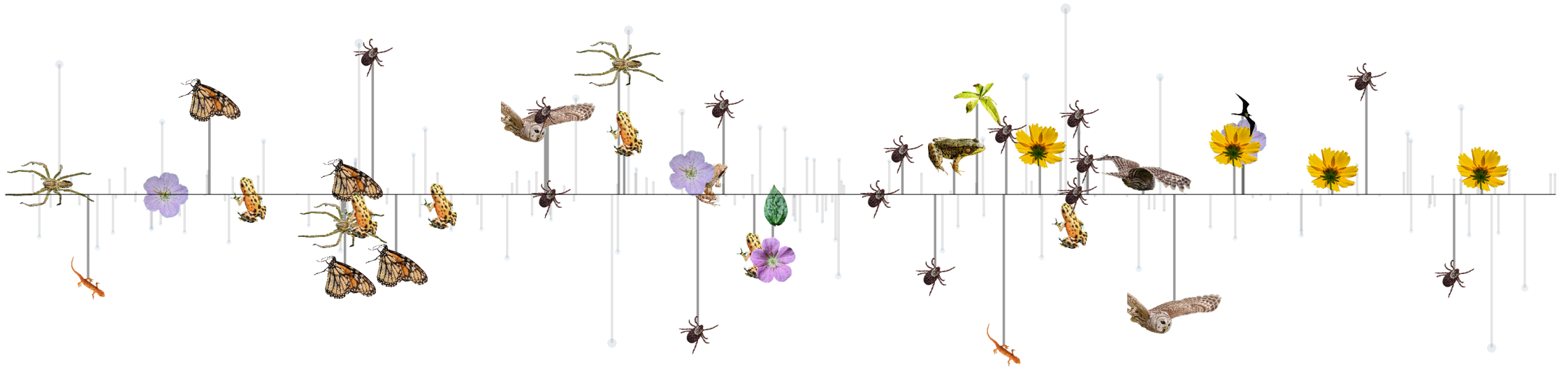


Intro to Quantitative Ecology

UMass Amherst – Michael France Nelson

Deck 13 – Potpourri: Spatial Ecology, Probability



Spatial Ecology

A Brief Intro

Important Concepts

- Composition and configuration
- Point processes and point patterns
- Complete spatial randomness
- Poisson distribution
- Basic probability theory

Probability Theory Essentials

Probabilities are non-negative

- A probability can be any value between zero and 1.0, inclusive.
- The probability of a specific event is usually less than 1.0
- **Law of total probability:** The sum of the probabilities of all possible events is 1.0

Sample space: the set of all possible events

- Events: a possible outcome of a stochastic process
- The definition of event is context specific:
 - “What is the probability of catching a fish that weighs 405 grams?”
 - “What is the probability of catching a fish that weighs between 399 and 411 grams?”
 - “What is the probability of catching a fish that weighs less than 200 grams?”

Probability Notation Basics

Basic probability

- $\Pr(A) = 0.05$
 - Read as: “The probability that event A occurs is 5%”

Joint probability

- $\Pr(A \text{ and } B) = \Pr(A \cap B) = 0.05$
 - Read as: “The probability that both events A and B occur is 5%”

Conditional Probability

- $\Pr(A|B) = 0.05$
 - Read as: “The probability that event A occurs, given that B has already occurred is 5%”

Independent Events

$$\Pr(A \cap B) = \Pr(A) \times \Pr(B)$$

- The probability that A and B both occur is equal to the product of the individual probabilities...
- We'll dissect this surprisingly important definition.



Independent events

Events are independent if knowing the value of one observation gives us no information about the value of another observation:

1. I measure the temperature in Neuquén, Argentina on November 23, 1823.
2. I measure the temperature in Amherst on July 4, 2020.

The Neuquén temperature in 1823 probably doesn't tell me much about Amherst in 2020

- Likewise, the temperature here today probably won't tell me much about what to expect there! [Other than knowing that it is fall here, and spring there!]

Independent events

Non-Independent Temperatures

Compare the previous temperature example to:

1. I measure the temperature in Amherst on July 4, 2020 at 4:05PM (it is 20C)
2. I measure the temperature in Amherst on July 4, 2020 at 4:11PM (it is 21C)

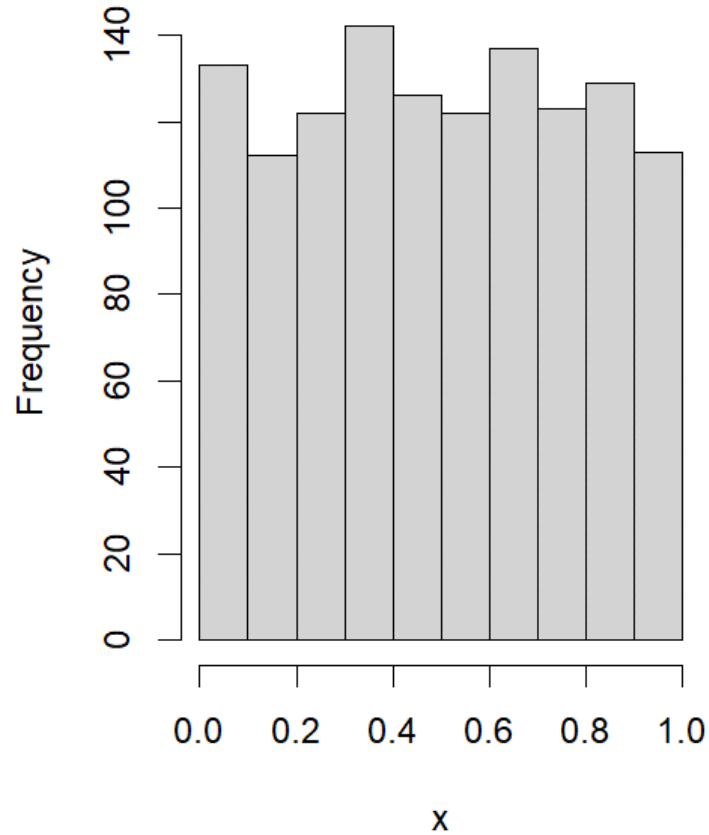
The temperature at 4:05 gives me a lot of information about what the temperature will be in the same location six minutes later.



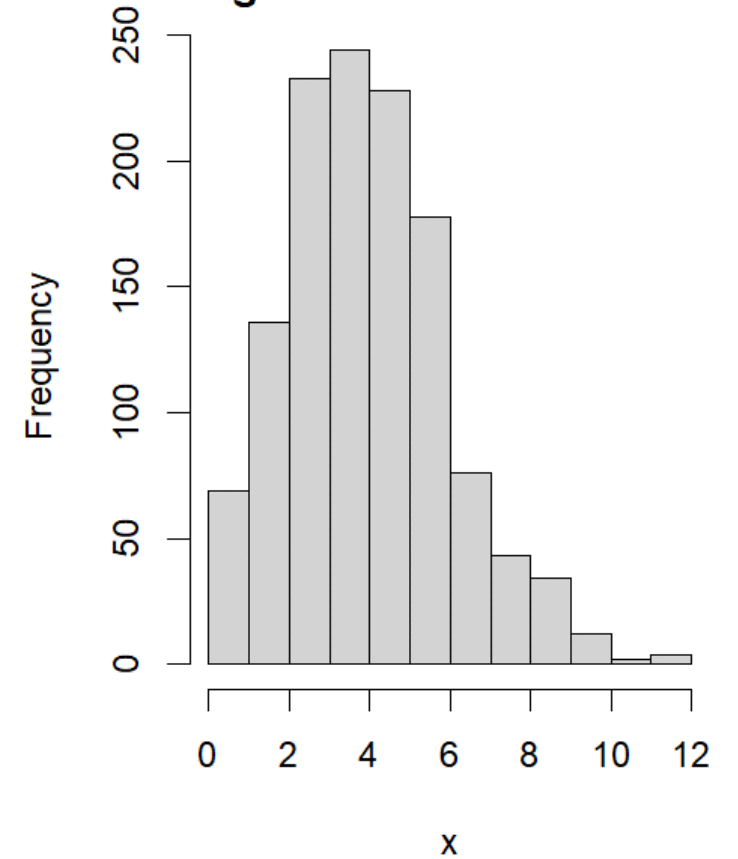
Poisson and Uniform Distributions

- Deep connection between Poisson and uniform distributions.
 - But first, let's learn about both!

Histogram of Uniform Numbers



Histogram of Poisson Numbers



Distribution Terminology

“A Probability Distribution Function maps every element in the sample space to a relative measure of likelihood”

- But what does that mean??? Let's break it down:

Sample Space:

- The set of all possible observations, also known as events, for a distribution
- All possible x-values

Relative Likelihood

- The height of the Probability Distribution curve for a given value of x

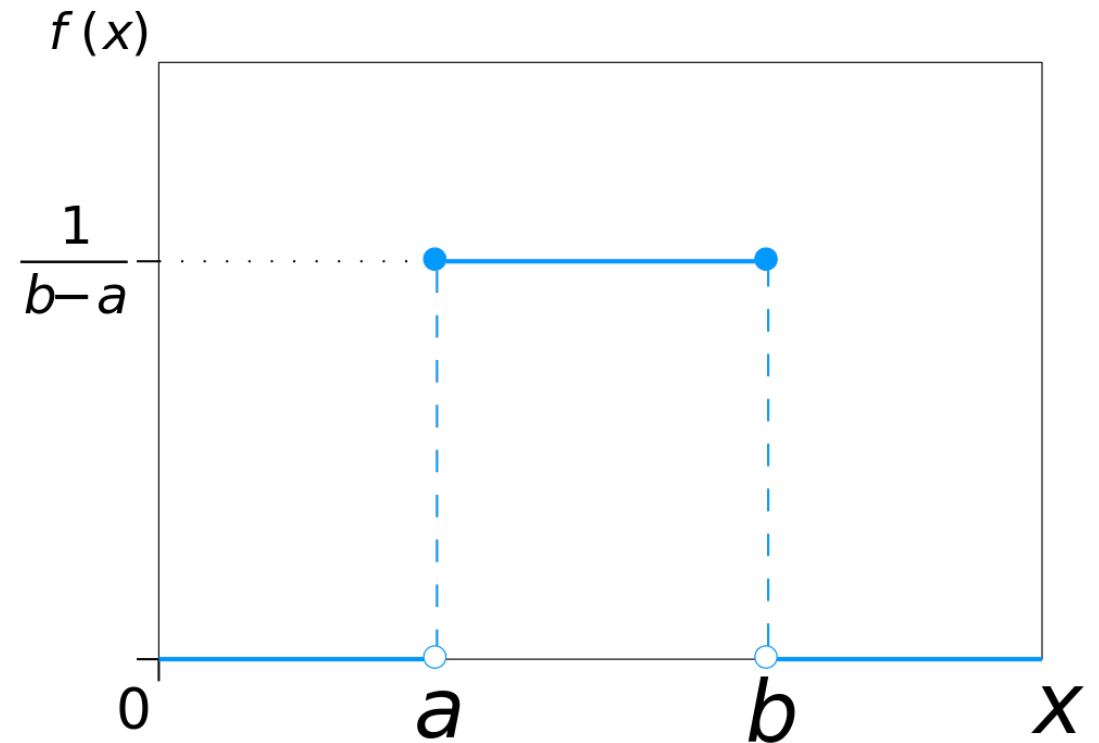
Distributions We've Met

So far, we've discussed:

- **Normal distribution:** hump-shaped, symmetrical
- **T-distributions:** Finite-sample version of the Normal, occurs in the t-test and linear regression
- **F-distribution:** Ratio of two variances, occurs in ANOVA and linear regression.
- **Chi-squared:** Sum of normal distributions, occurs in the chi-square test of association.
- **Binomial and Bernoulli distribution:** counts of successes in n-trials. Coin flips, numbers of plots with presence/absence

The Uniform Distribution

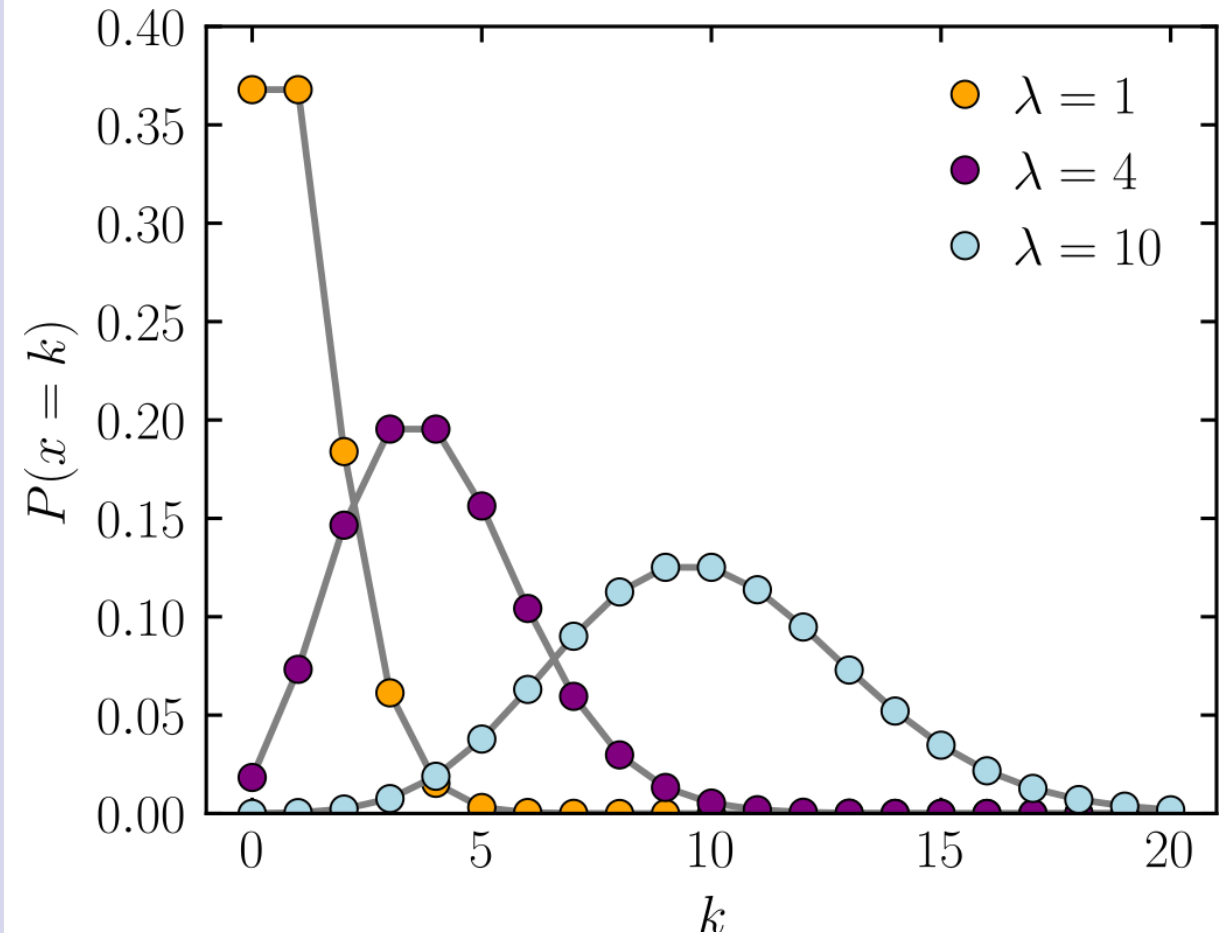
- The Uniform is one of the simplest distributions.
- It maps every event in the sample space to the same probability
 - What is the probability of observing a fish measuring 34 millimeters?
- An interval event from a to b has probability $\frac{1}{b-a}$
 - What is the probability of observing a fish between the lengths of 34 and 37.5 millimeters?



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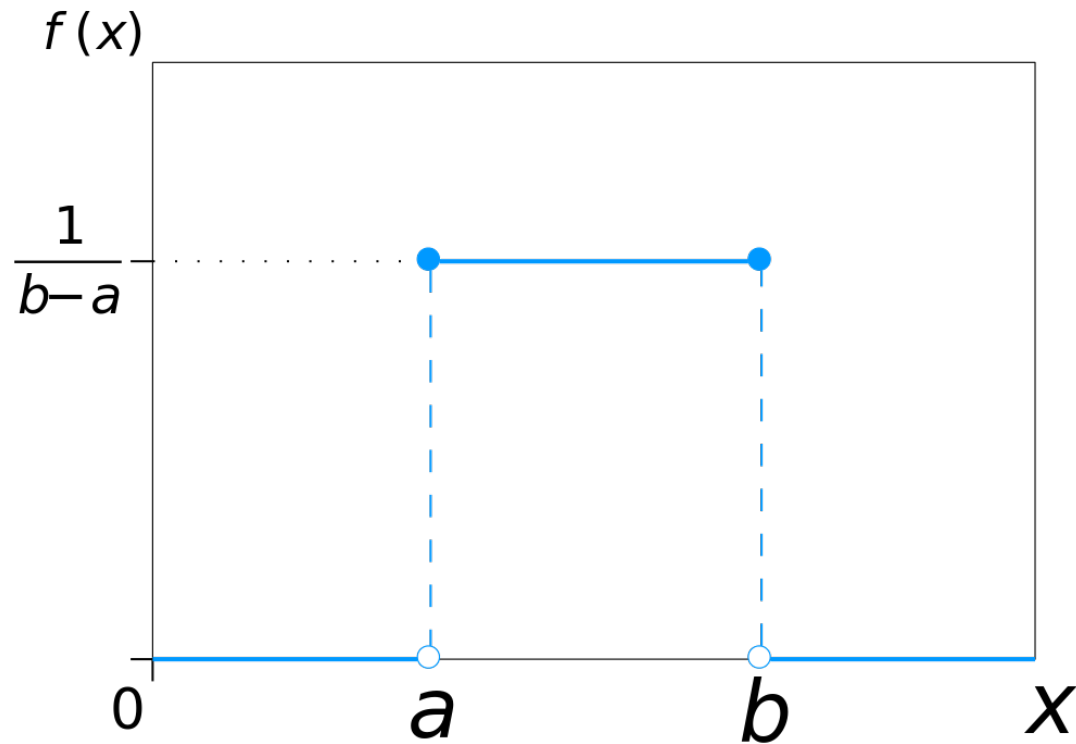
The Poisson Distribution

- The Poisson is a 1-parameter, discrete distribution.
- λ is equal to both the mean and standard deviation.
 - Remember how the mean and standard deviation were independent for the Normal distribution? This is not the case for most other distributions.
 - Lambda determines the shape of the distribution: different values have different amounts of skew.
- Poisson distribution models events that occur at a constant (uniform) rate in space or time.

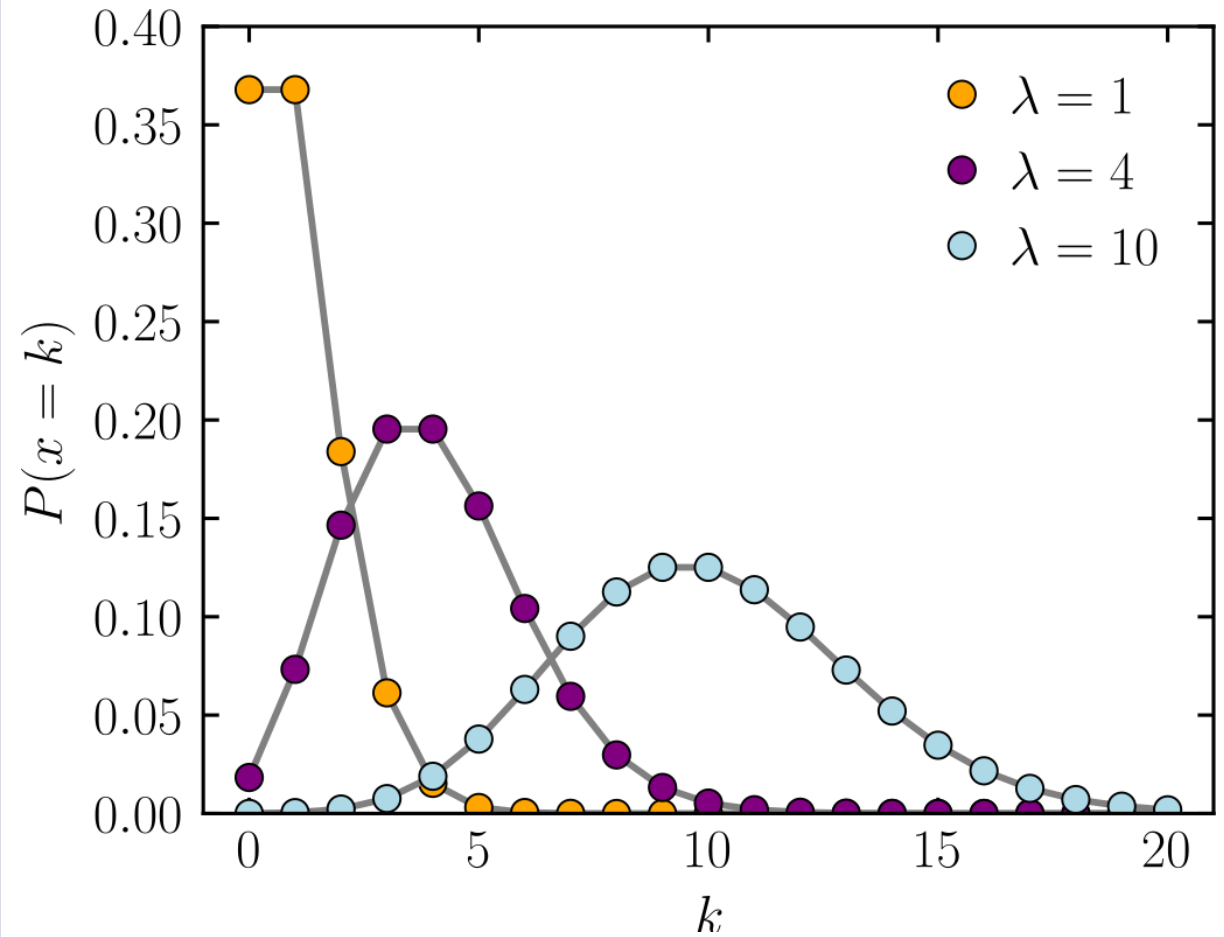


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What's The Connection???



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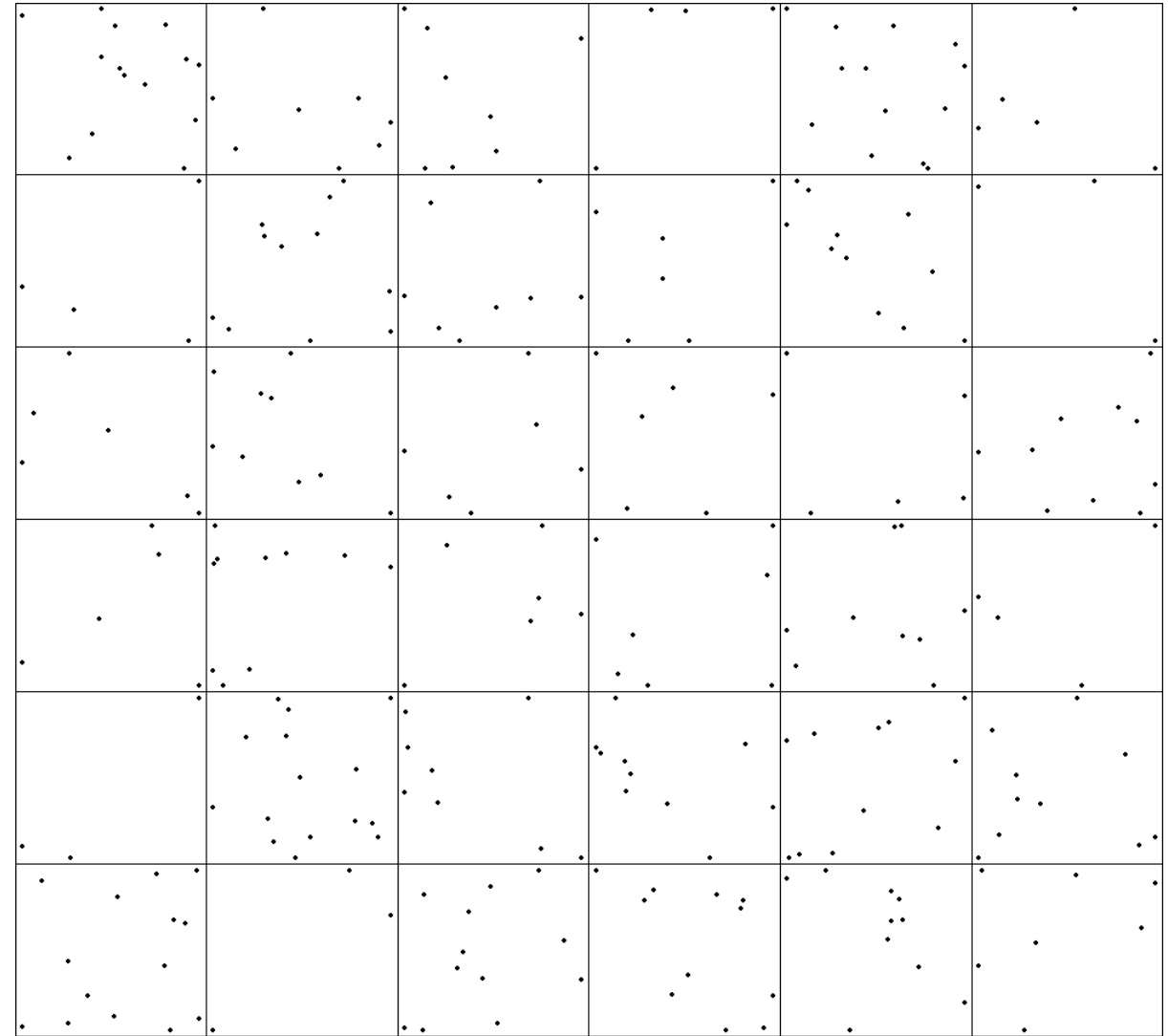


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What's The Connection

If you sample x - and y -coordinates from a **uniform distribution**, the number of points within equally-sized sub-sections are **Poisson-distributed**.

- We've gone from one distribution to another
- We've transformed a continuous distribution, the uniform, into a set of counts described by a discrete distribution, the Poisson!



Why Should I Care?

You may be thinking...That don't impress me much!

- But think about it a little bit, how might this be useful to us?
- Could we use this as a null model?
 - What about our desert shrubs example?

Point Patterns and Processes

Pattern and Process

Processes generate patterns

A realization of a stochastic point process generates a point pattern.

Poisson point process

- Definitions
- Differences
- Inference

What's a point process?

- Any process that occurs in space that results in discrete positions of organisms, events, objects, etc.

What's a point pattern?

- The resulting set of locations of organisms, events, objects, etc.

Configuration

Point patterns are all about configuration!

Configuration refers to the physical positions of entities in the landscape.

For example, points may be arranged randomly.

Configuration vs. composition

Composition is the relative proportions of different types of entities in the landscape:

- Proportion of conifer/hardwood forest
- Amount of agriculture in the landscape

Configuration is the relative position of different types of entities in the landscape?

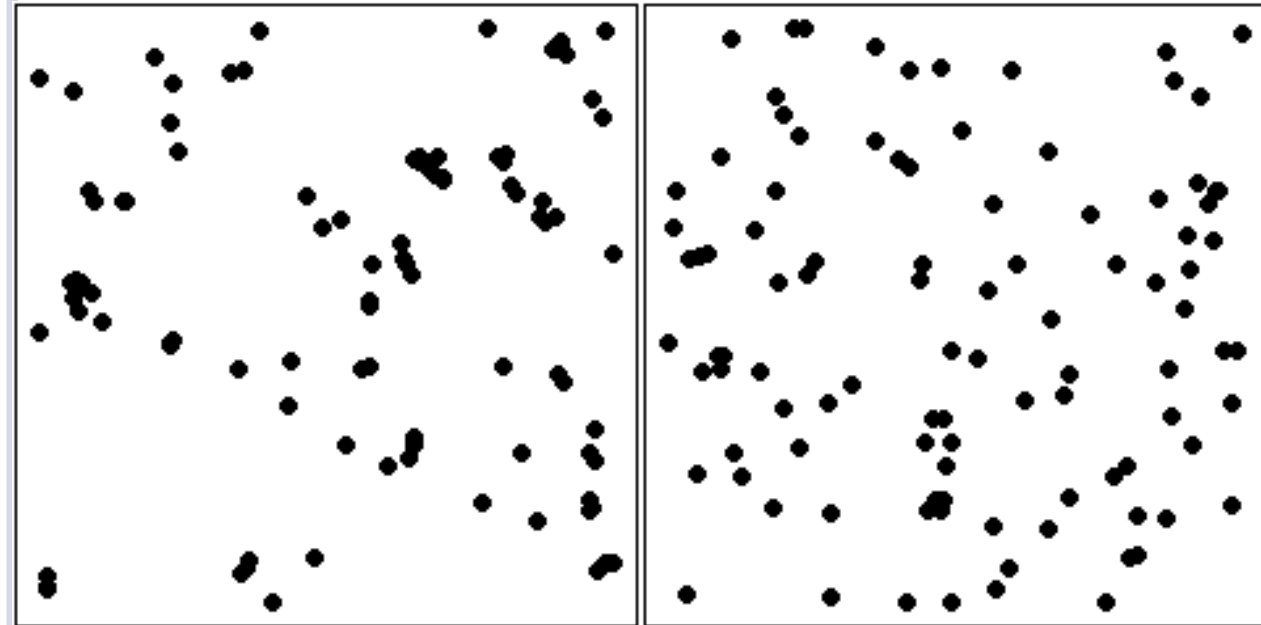
- Patch size, patch arrangement

Landscapes with the same composition can have different configurations.

Configuration of point patterns

How would you describe these patterns?

How could we quantify the differences in the patterns?



Distance: Distance Matrices and Nearest Neighbors

Distance Measures

We can represent pairwise distances between spatial objects (like points) using *distance matrices*.

Distance matrices are square with dimension n , where n is the number of spatial objects.

Distance matrices have n^2 elements – they can get very large for large datasets

We choose what type of distance to include.

- Euclidean, Manhattan, neighborhood, network

Neighbor Measures

We can calculate the distance to the nearest neighbor.

We know how the nearest neighbor distances should be distributed in CSR.

- We could compare the observed vs. the expected... This sounds like inference!

Types of Point Patterns

Complete Spatial Randomness (CSR) and Clusters

Complete Spatial Randomness

The x , and y , (and z for 3D space) coordinates are distributed *independently* and *uniformly* within the boundary.

What use is CSR?

Cluster Patterns

Matern process

- Parent points are CSR, offspring points occur near the parents
- Results in a 'clustered' pattern

Poisson Distribution

Poisson distribution: 1-parameter, lambda, distribution

- The lambda is the *center* and the *spread*.
- Variance equals the mean.

What is one key difference from the normal distribution?

Poisson and uniform distributions in CSR

- Points are uniformly and randomly distributed in space.

Create a Poisson pattern

One way to create a Poisson point pattern:

- Define your intensity, a region of interest (ROI), and calculate the area of your ROI.
- The Poisson parameter (for the entire ROI) λ is *intensity* \times *area*
- Generate a Poisson-distributed random number, n
- Generate n independently and uniformly-distributed x - and y -coordinates for the points.

What properties would such a pattern have?

Why is it called a Poisson pattern if we use the uniform distribution?

- Deep connection between uniform rates and the Poisson distribution.

Poisson process and pattern

Number of points in independent samples (of the same area) are Poisson-distributed.

Link between the uniform and Poisson distributions!

Point counts are Poisson-distributed regardless of the area of your samples!

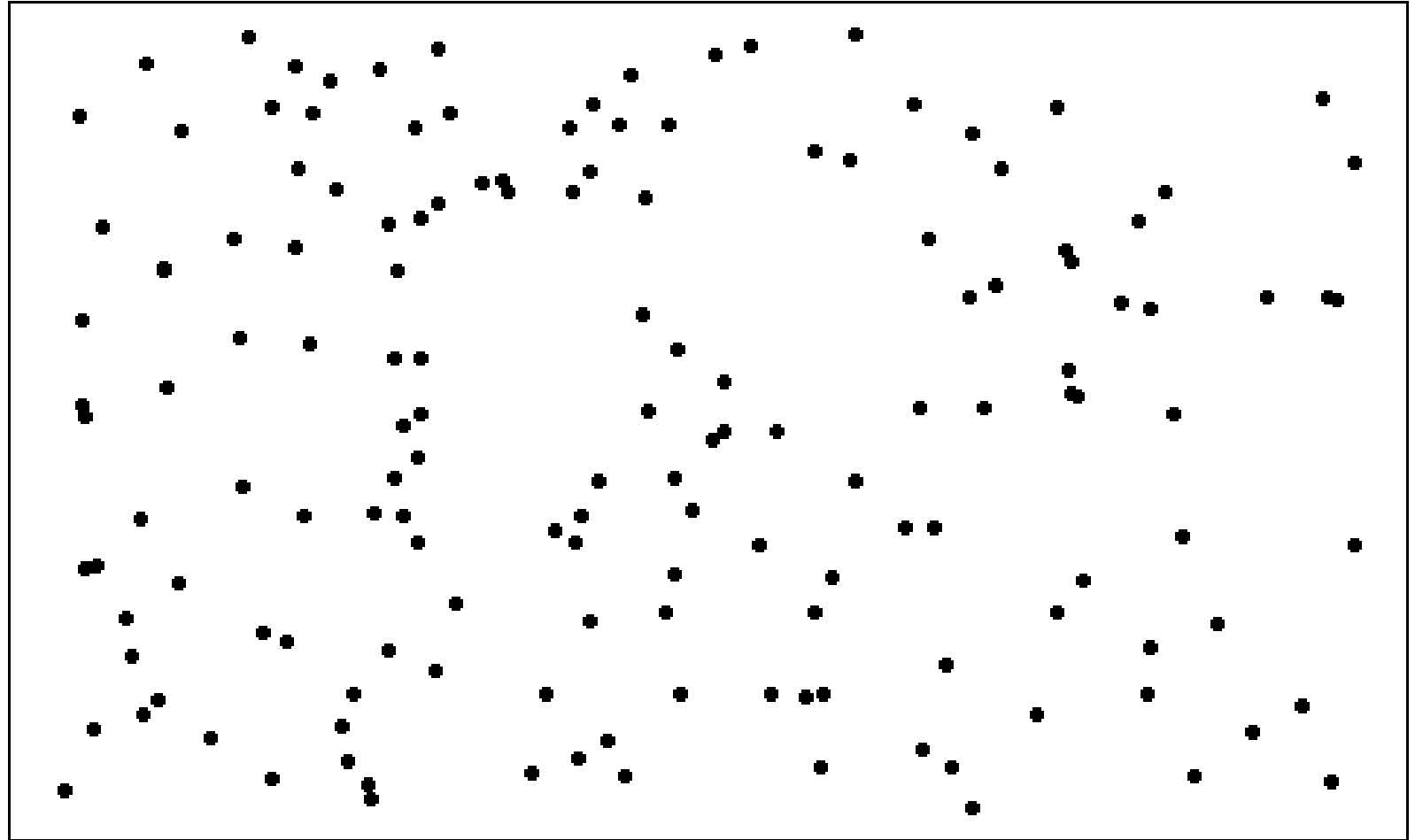
- But... what changes if you use different-sized samples?

When I first heard about this, I had to 'prove' to myself that this is true with some simulations. [Poisson process demo R script]

Random Is Clumpier Than You Think

Should *random* feel somewhat evenly spaced?

I was surprised by how clumpy randomness can look:



Over- and underdispersion

I had imagined an *overdispersed* pattern.

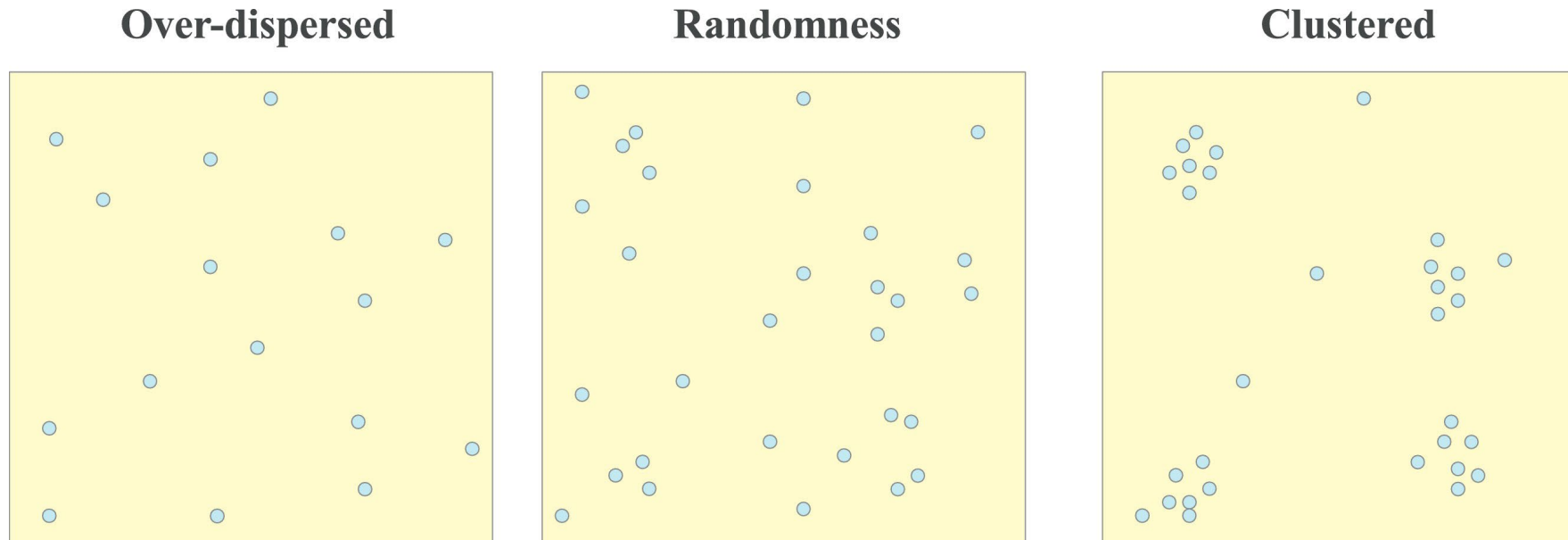


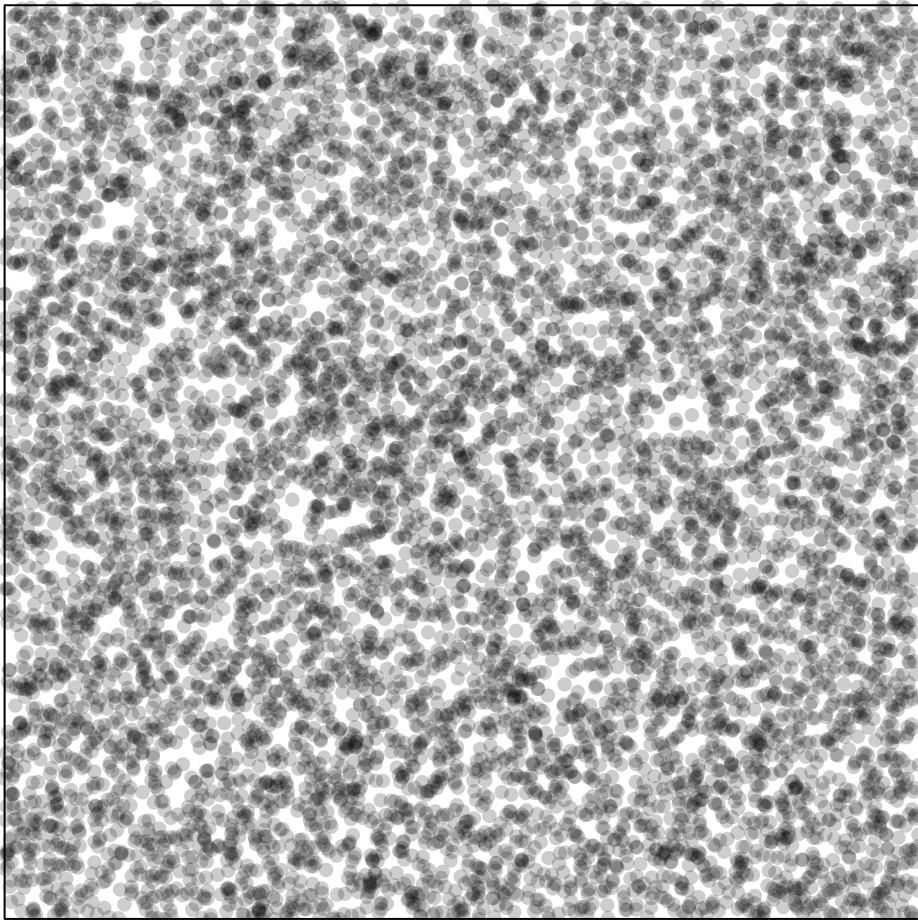
Figure 4 in Abraham et al. 2013 Modern statistical models for forensic fingerprint examinations: A critical review

What do Point Patterns Look Like?

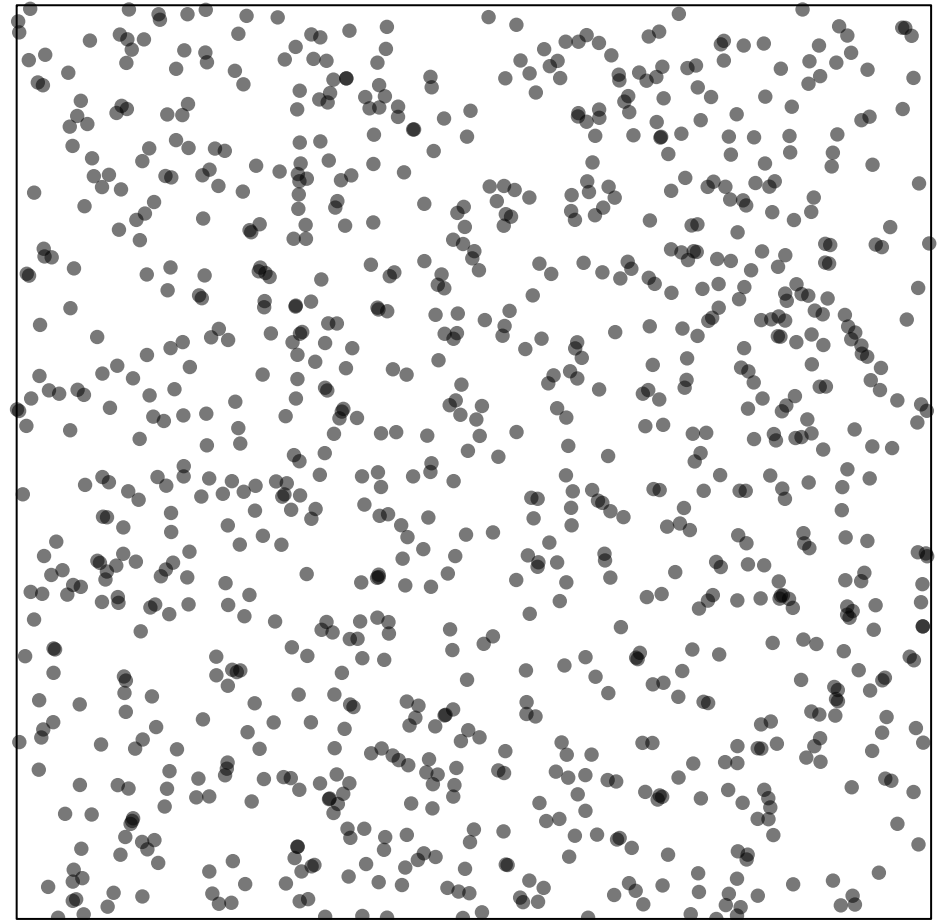
CSR and Matern

Complete Spatial Randomness

CSR, $\lambda = 100$

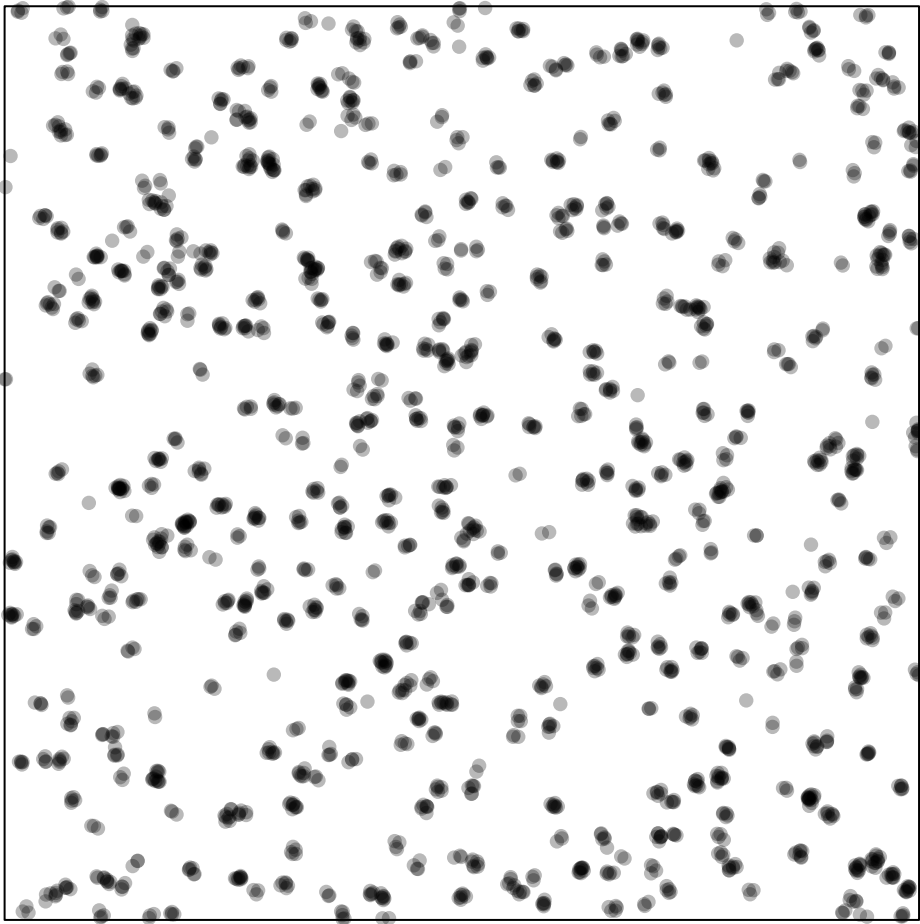


CSR, $\lambda = 10$

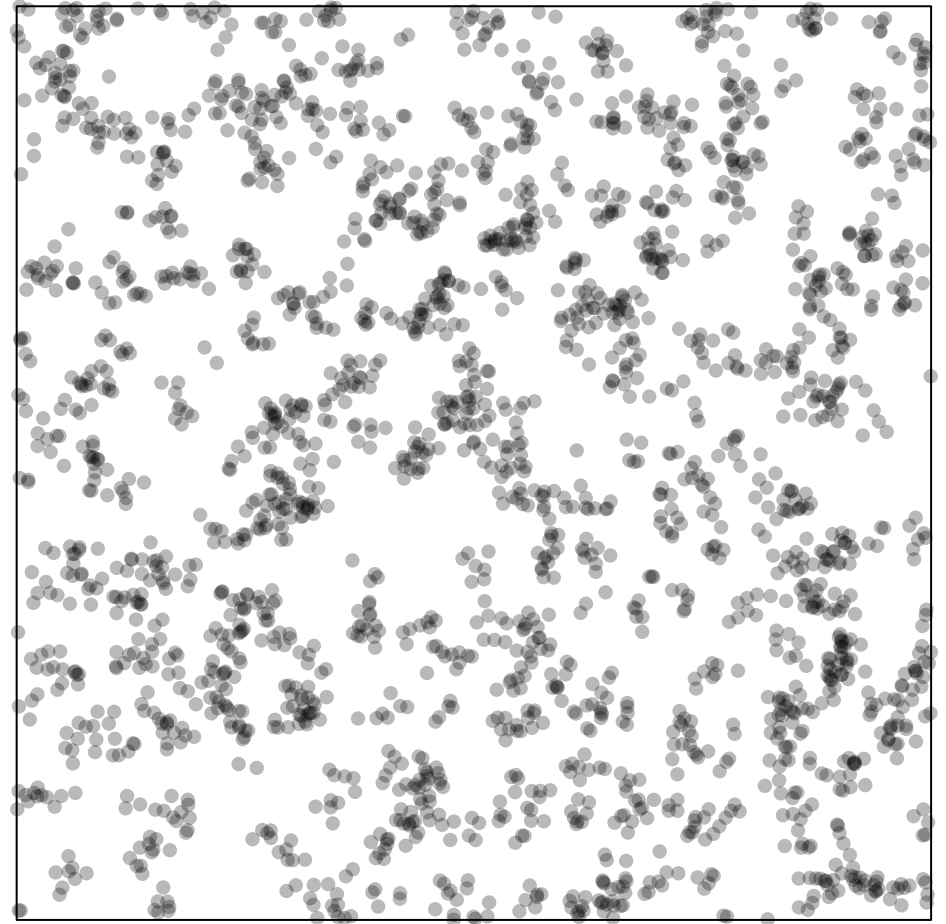


Matern Cluster Patterns

Matern: $k = 5$
scale = 0.05, $\mu = 5$

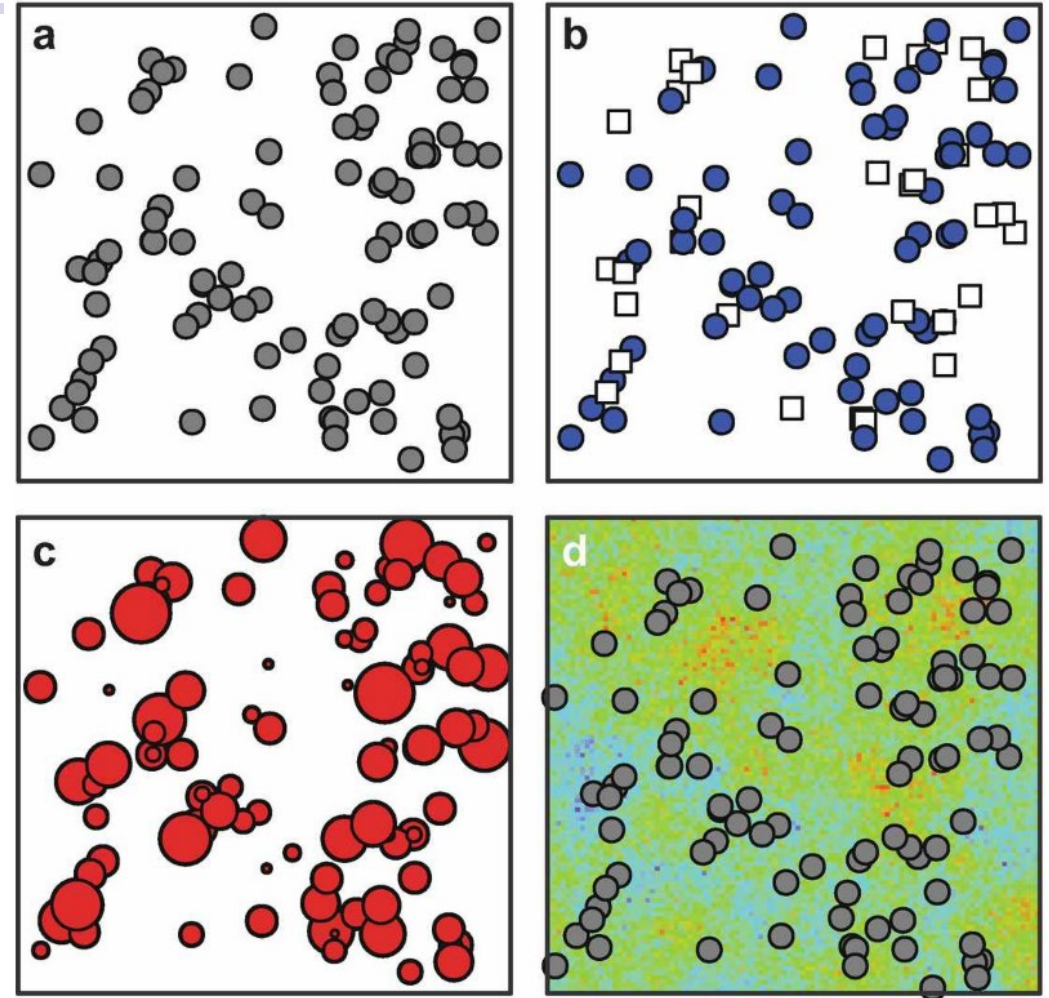


Matern: $k = 5$
scale = 0.2, $\mu = 5$



Marks

- So far, we've only talked about unmarked, or univariate, point patterns.
- Points can have characteristics (marks)
- For example:
 - Species (categorical)
 - Tree diameter (numeric)
 - Presence/absence
 - Elevation
- Points can have multiple marks



F + F **Fig. 4.2** Some common characteristics of point patterns.

Interpreting Point Patterns

Recall the definitions

Density-based measures

- How many points are within a circle of radius r ?

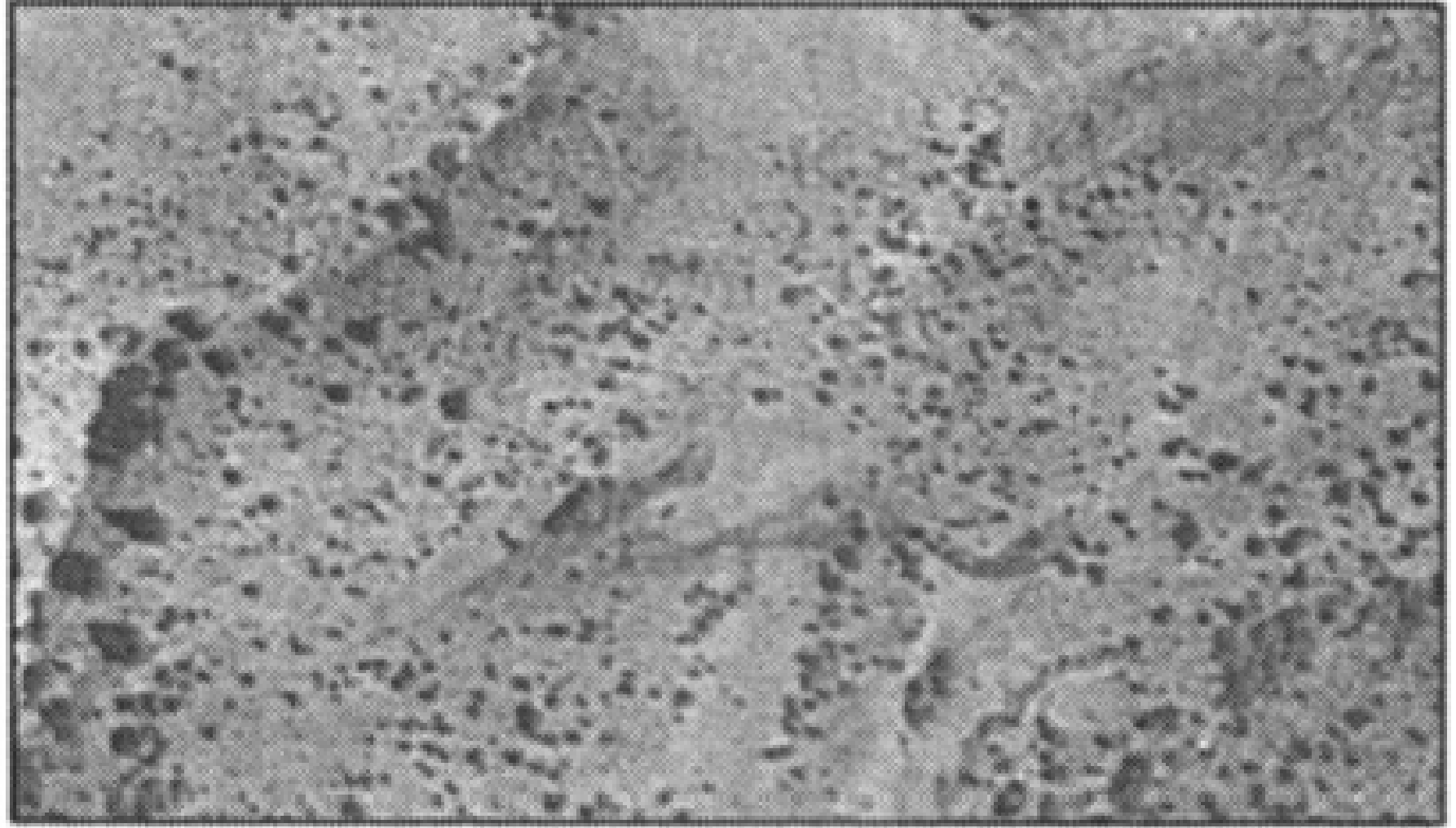
Neighbor based measures

- What is the probability that my nearest neighbor is within a circle of radius r ?

An Example: Sagebrush/Juniper Steppe

Spatial patterns on the sagebrush steppe/Western juniper ecotone

**Eva K. Strand · Andrew P. Robinson ·
Stephen C. Bunting**



Desert Shrubs and Point Patterns

More to come...

- We'll explore CSR and point patterns in greater depth on Thursday!



NEON Assignments

NEON slides

- Read your slide feedback
- Add a conclusions statement
- Get ready to present next Tuesday!
- Are you in the default group? If so, you need to join a group before we can grade your submission and provide feedback!

In-Class Assignment

- Submit your group's top 3 questions