Analysis of Environmental Data

Data Exploration, Associations, and Functions

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Announcements

- Updated Deck 3 slides: re-download!
- DataCamp: Intermediate R
  - Optional, but may be useful as a supplement to the course/lab materials
- In-Class Model Thinking
  - Graded, except for those in the Default Group
- Default Group: If you don’t have a grade, make sure you’re part of a group (even if it’s a group of 1).
- Azure virtual desktop…. If you have problems logging in, create a screenshot and include an explanation of what you tried to do.

Understanding variables vs. functions

- **Variables and functions both have names that you type into r without using quotation marks.**
- **Functions are typically evaluated, and they return a result of some type.**
- **Functions may return objects like vectors, matrices, data frames etc.**

Variables	Functions
• Can contain any kind of R object • Single number, vector, data frame, etc • The class() function will tell you what kind of object they hold • They are like nouns in that they don’t perform an action, they just represent an object. • Variables aren’t followed by parentheses. • We can assign the output of a function to a variable	• Functions are a particular kind of entity in R. • We type parentheses at the end of a function name to let R know that it’s a function and that we want to evaluate it. • Functions are like verbs; they may take an object, and they perform some kind of action, possibly returning a value.

Evaluating a function + saving to a variable

- When we call a function in R, it may return a value or object.
- We can assign the **function output to a variable**.
- For example, in the **expression**:

```
m1 = matrix(1:6, nrow = 2)
```

- First, **matrix()** (a function) is **evaluated**, then **assigns the output** to **m1** (a variable)
- The material to the right of the assignment operator is always evaluated first.

What’s In This Deck?

Slides	Selected Key Take-Home Concepts
• Data Exploration • Types of Plots • Functions, variables, constants • Formulae and notation • Classes of functions • Intro to distributions	• Figuring out which parts of a function are variables, and which are constants. • Bases vs. exponents • Exponentials win over powers every time! • Linear, asymptotic, and monotonic. • Summarizing and raw-data plots.

Data Exploration

With examples in built in R!

* R code available on request, absolutely no warranty.

Statistics and Parameters: Frequentist Perspective

We're guests in a Frequentist world	[MS Office Art Suggestion]

- Let's think about data exploration from a Frequentist perspective!
- What is a statistic and what is a parameter?
- What is a population and what is a parameter?

Data Exploration

Numerical	Graphical

• Compact summary of data
• Extremely important, but not as intuitive as a graphical exploration
• Summary statistics:
    • Center
    • Spread
    • 5-number summary
	• Helps you get an intuitive ‘feel’ for what’s in your data.
	• Graphs/Plots!
	    • Many types, each shows different aspect of data.
	• Important distinction: does my plot show all data points, or a summary of aggregated data?

Data Exploration

We're most often interested in two characteristics of our data:	
Center	**Spread or Dispersion**
• Mean • Median • Mode	• Range: min and max • Interquartile range (IQR) • Variance • Standard deviation

Data Exploration

- Center and spread are easy to understand numerically.
- Other quantities make more sense graphically:
  - Skew
  - Kurtosis
  - Bi- or multi-modality

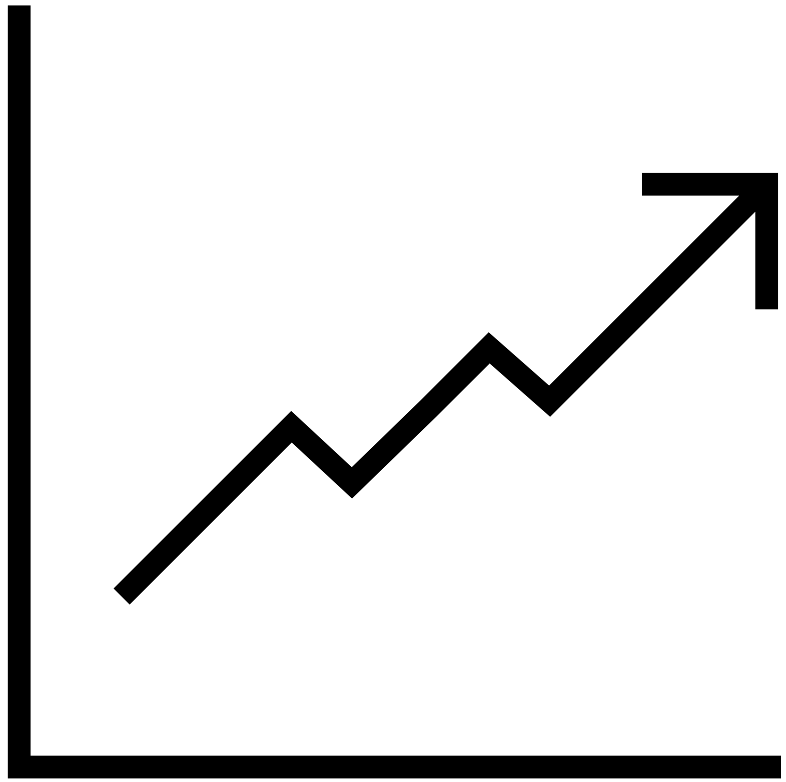
Associations

***Association* is a value neutral term.**

- It is useful when you don’t want to imply causality, or any specific form of a relationship.

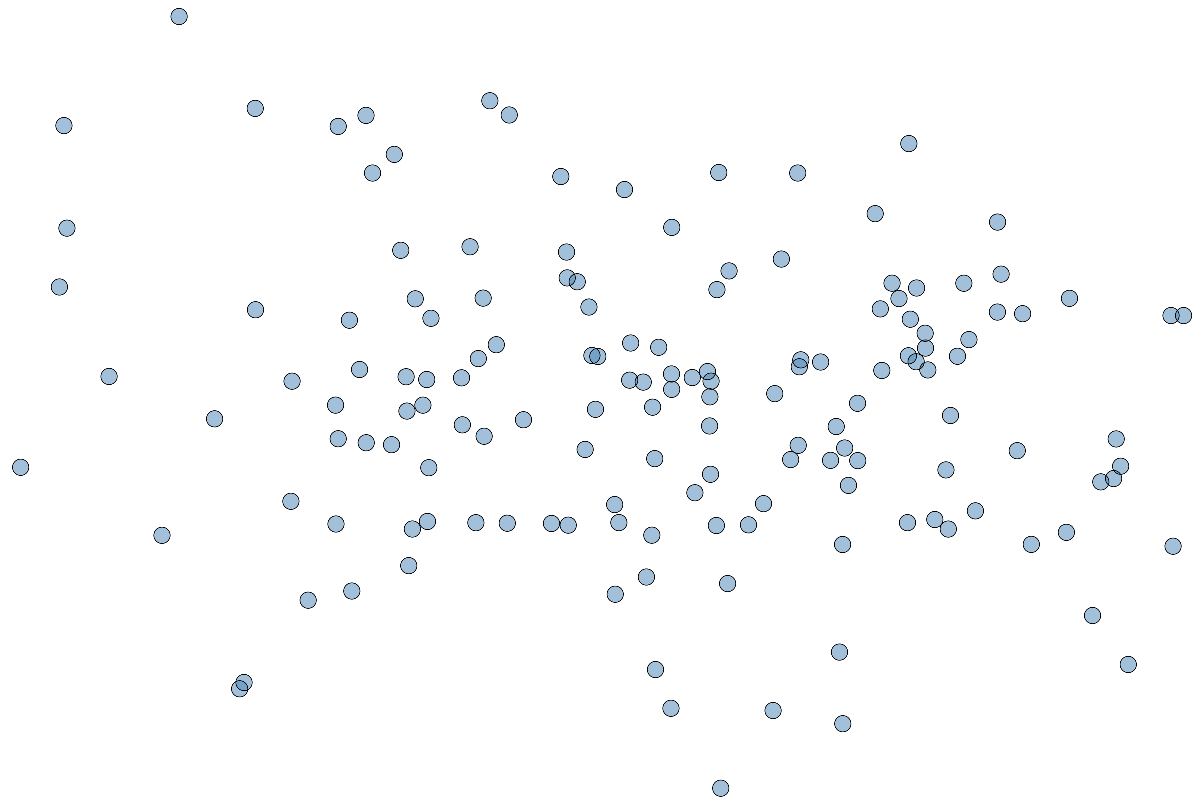
How can we *describe* an association?

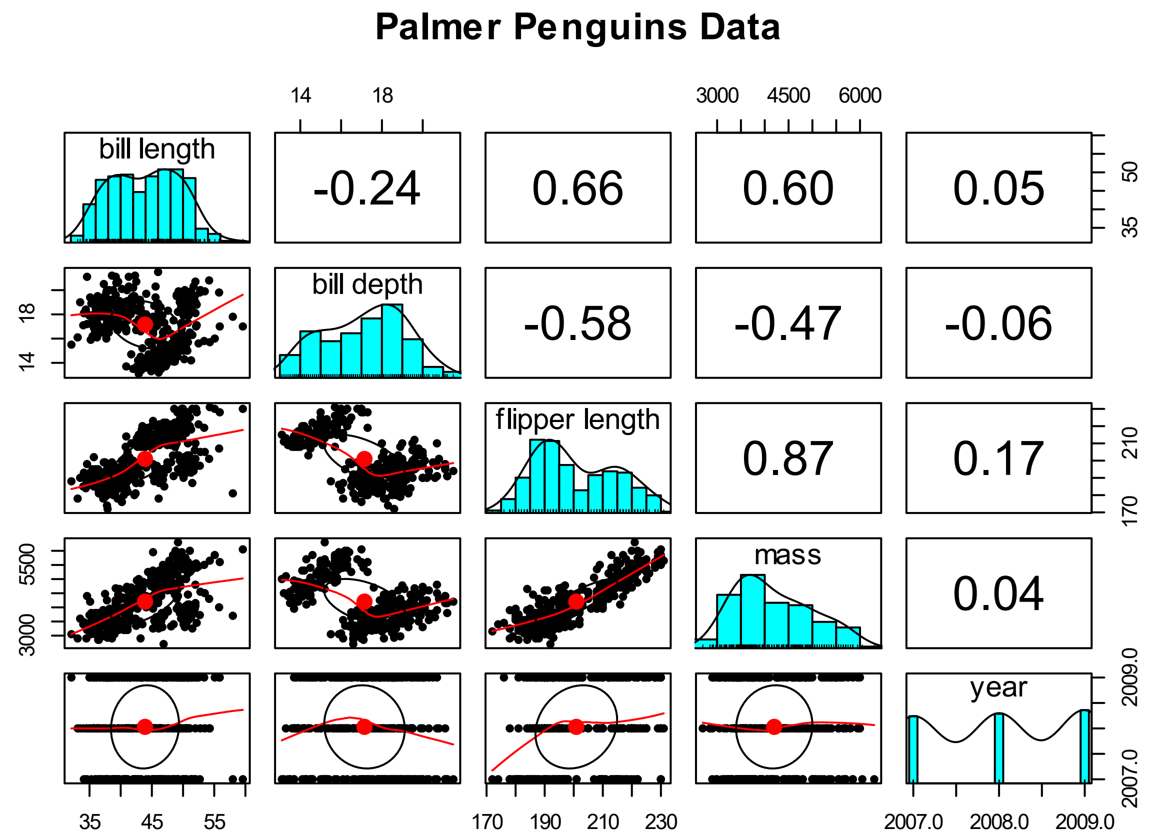
- Qualitative and quantitative
- Numerically and graphically



Associations: graphical exploration

Scatterplots are useful	Pairplots are even better!





Correlation Coefficients

Correlations describe the strength of association between two variables	Some limitations and caveats

• Correlations measure how close points lie to a curve.
• How well can you predict y from x?
• Correlations are a kind of descriptive stochastic model.
    • But not a very powerful one, as we'll see.
	• Limited to **two** variables.
	• Spearman and Pearson correlations are limited to **monotonic** functions.
	• Does not tell us anything about the **magnitude** of an association.
	• Cannot deal with **multi-collinearity**.
	    • But don't worry, we have tools that can.

Correlation

Correlation Measures the Strength of the Association Between two Variables.

- Correlation ranges from -1 to 1:
- 1 indicates **perfect correlation**
  - Bivariate data lies exactly on a line of positive slope
- -1 indicates **perfect negative correlation**
  - Data lies exactly on a line with negative slope
- 0 Correlation: Points are **totally random** with respect to each variable.

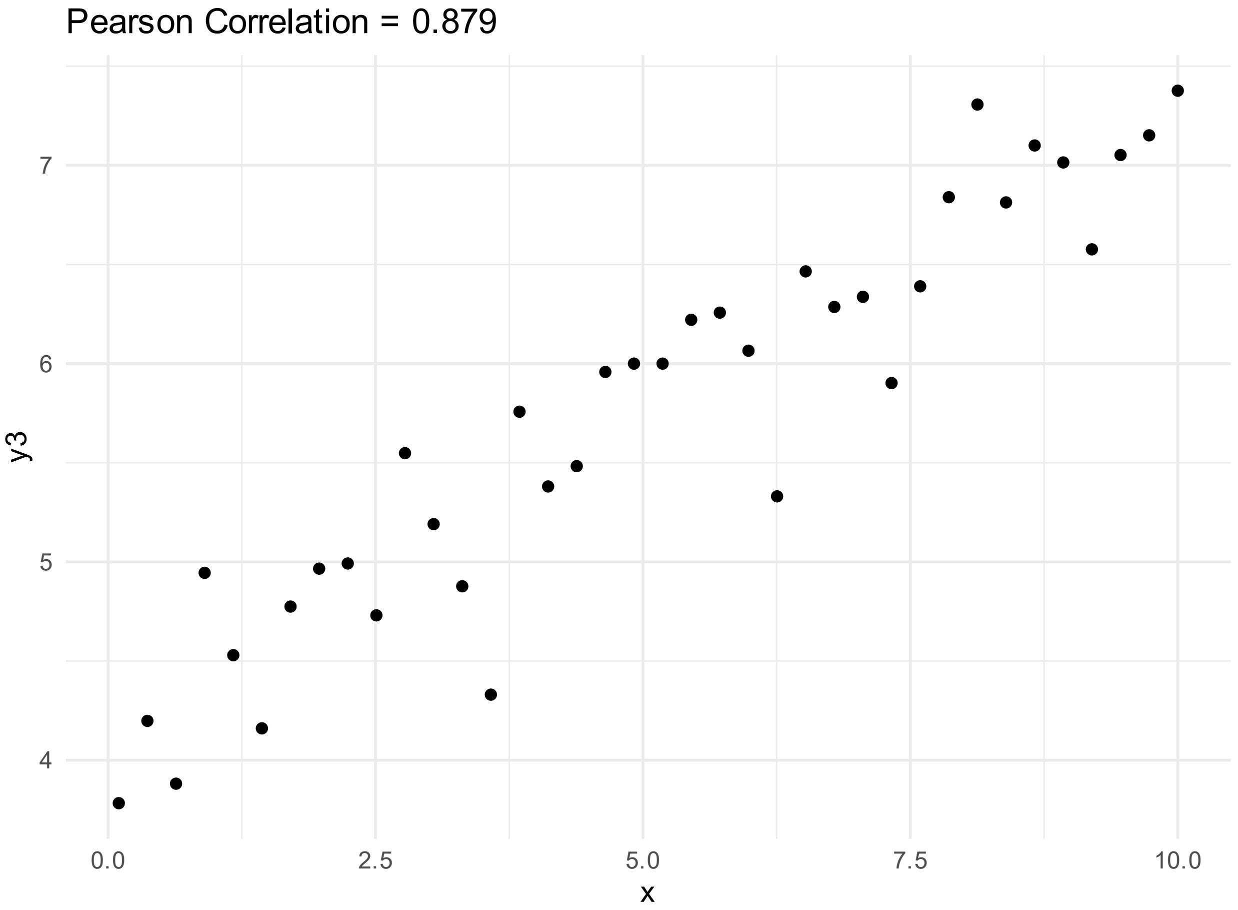
Correlation – Information Perspective

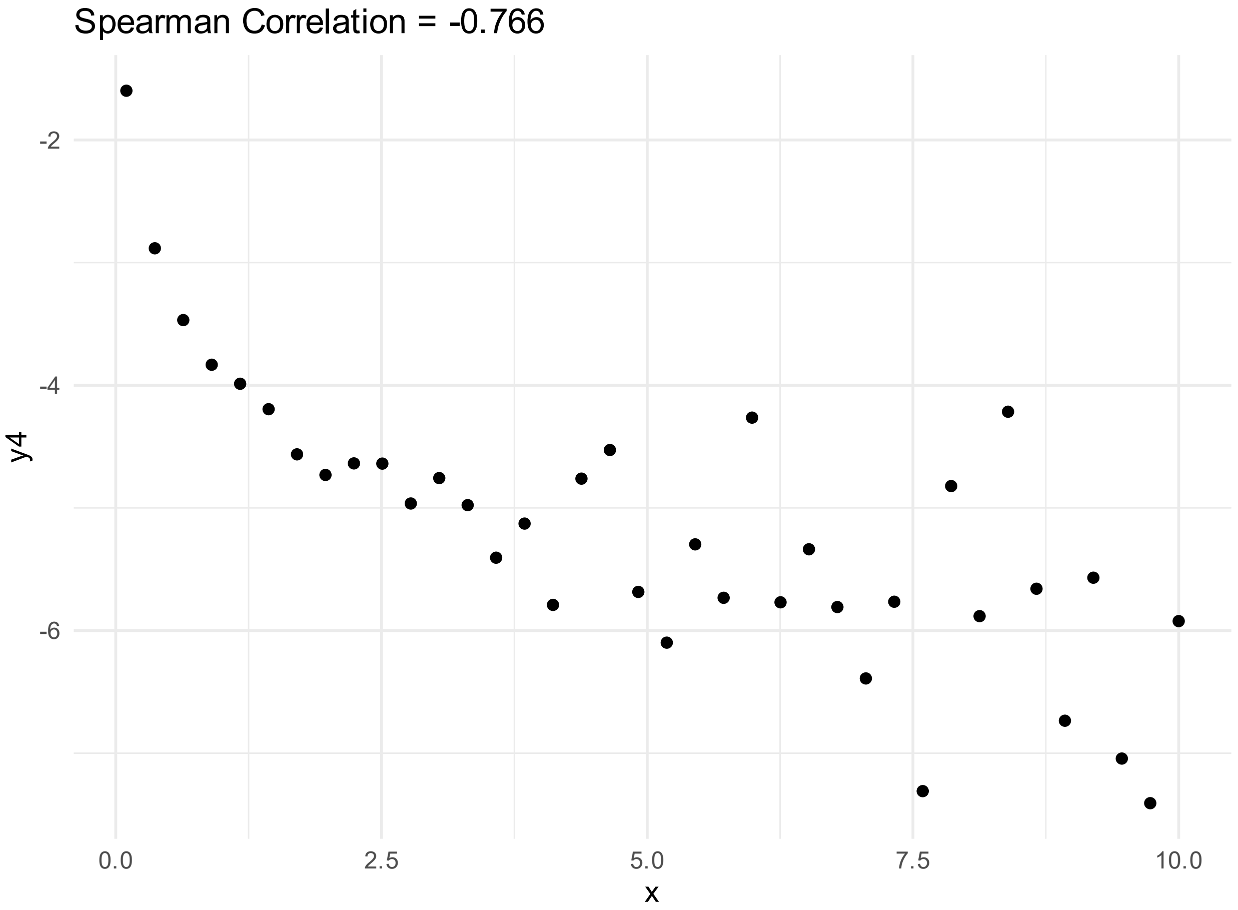
Correlation Measures the Strength of the Association Between two Variables.

- Correlation ranges from -1 to 1:
- Corr 1: We can predict y from x perfectly. There's no noise, and as x increases, y increases. x tells us all we need to know about y.
- Corr -1: We can perfectly predict y from x, there is a negative relationship.with negative slope
- 0 Corr: X tells us nothing about y.

Association: Numerical exploration

Quantifying linear association Pearson Correlation	Quantifying monotonic association Spearman Correlation



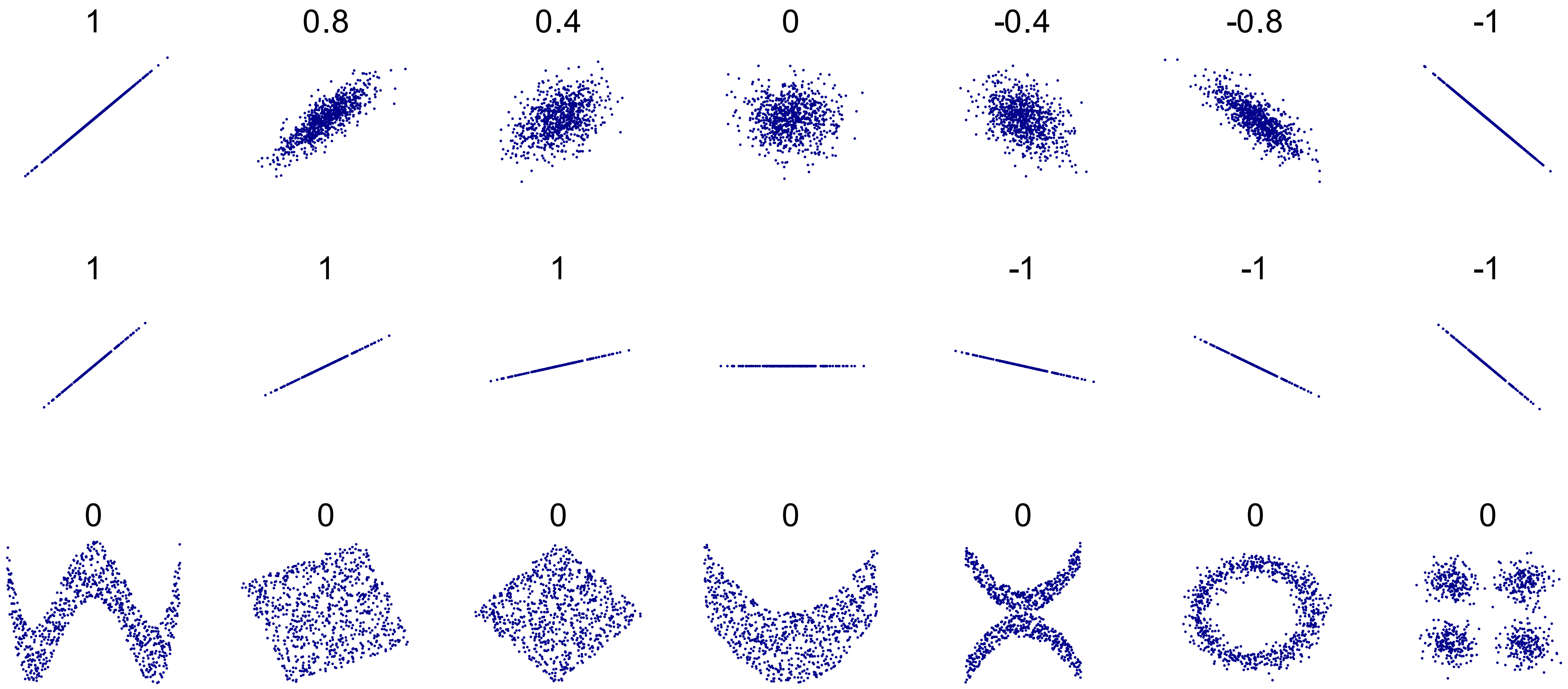


More complicated relationships

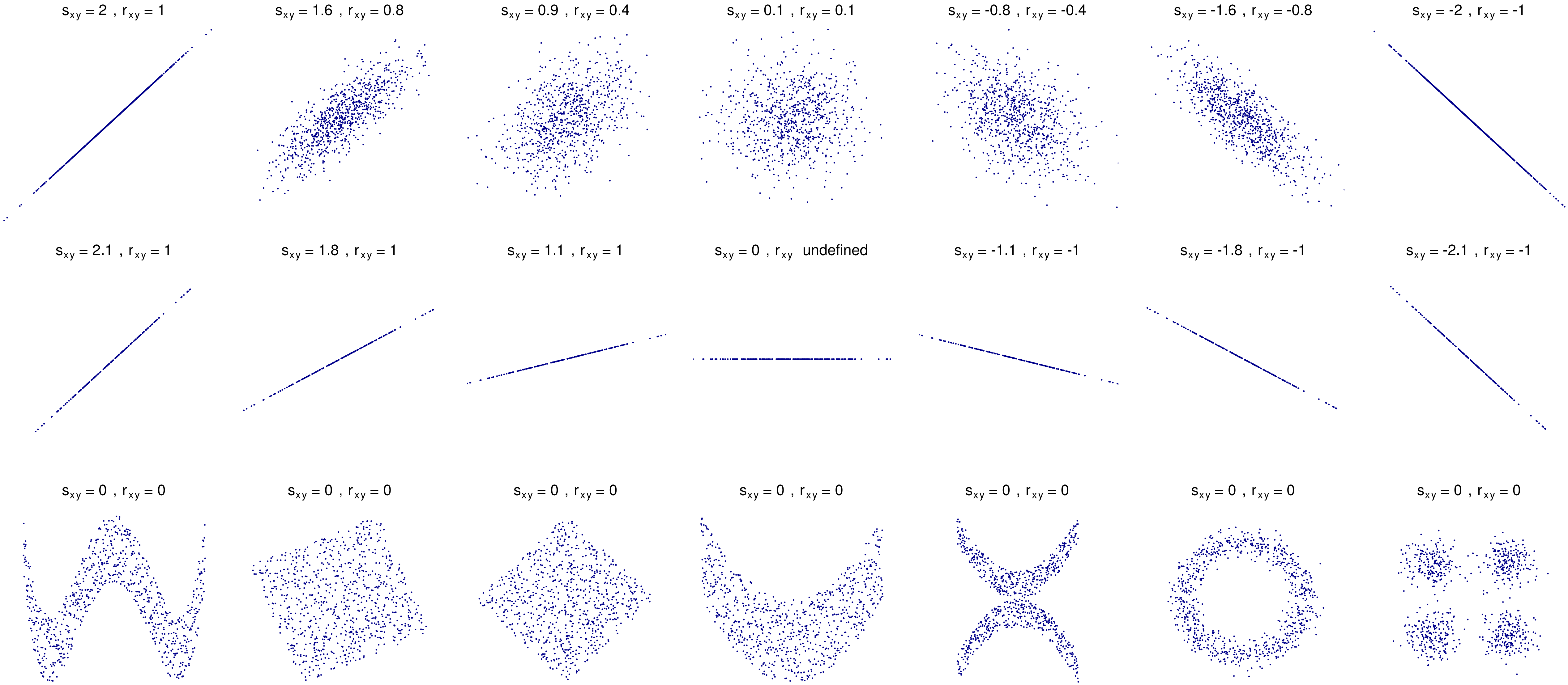
- How would you describe these associations?
- Does knowing the value of x tell you anything about y?

- Can you guess what the numbers (all 0) represent?

Correlation Coefficients



Denis Boigelot: public domain image


$s_{xy} = 2$, $r_{xy} = 1$
$s_{xy} = 1.6$, $r_{xy} = 0.8$
$s_{xy} = 0.9$, $r_{xy} = 0.4$
$s_{xy} = 0.1$, $r_{xy} = 0.1$
$s_{xy} = -0.8$, $r_{xy} = -0.4$
$s_{xy} = -1.6$, $r_{xy} = -0.8$
$s_{xy} = -2$, $r_{xy} = -1$
$s_{xy} = 2.1$, $r_{xy} = 1$
$s_{xy} = 1.8$, $r_{xy} = 1$
$s_{xy} = 1.1$, $r_{xy} = 1$
$s_{xy} = 0$, r_{xy} undefined
$s_{xy} = -1.1$, $r_{xy} = -1$
$s_{xy} = -1.8$, $r_{xy} = -1$
$s_{xy} = -2.1$, $r_{xy} = -1$
$s_{xy} = 0$, $r_{xy} = 0$
$s_{xy} = 0$, $r_{xy} = 0$
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$s_{xy} = 0$, $r_{xy} = 0$

In-Class R Practice

Announcements: Sep 20th

- Assignment File Formats: pdf or html (occasionally online text)
- Fixed correlation plot for colorblind accessibility (Deck 3)
  - Will work to update others as we go.
- First batch of assignments are graded! Check Moodle and let me or Ana know if there are any issues.
- Software setup assignment: If you didn't get 100%, you may resubmit. This is a special arrangement for this assignment only; it's vitally important to get the software setup correctly.
- In-class R practice feedback.

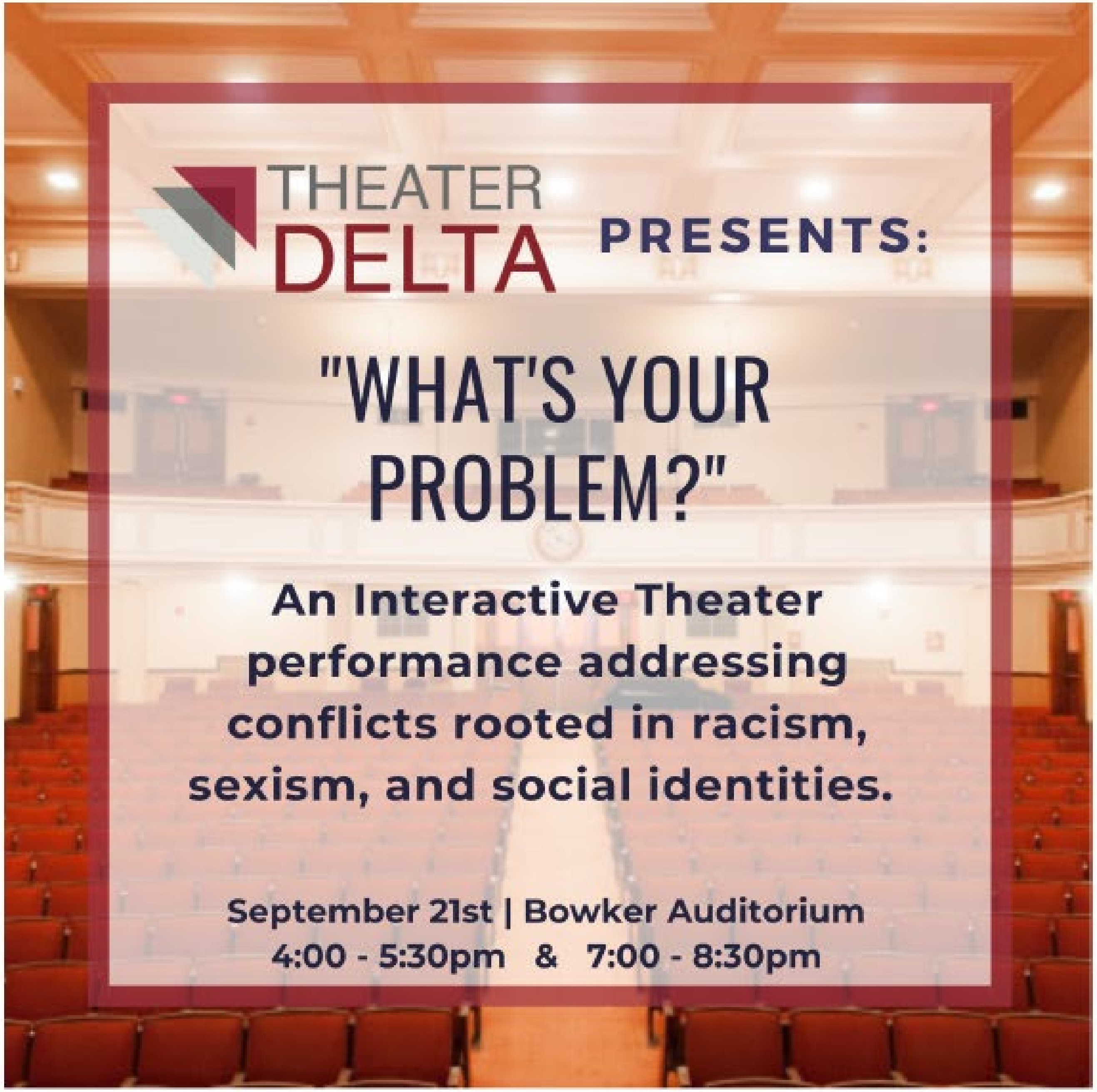
University of Massachusetts Amherst

JOIN THE ECO JUSTICE, EQUITY, DIVERSITY, INCLUSION, AND ACCESSIBILITY (JEDIA) COMMITTEE!

Please join our Leadership Team that meets biweekly throughout the school year, or a Task Force that only meets for a short time period to accomplish a task!

First meeting:
Friday, September 23rd, 11:30-12:30
312A Holdsworth and on Zoom

Email Meg Graham MacLean for more info
mgmaclean@umass.edu or diverseEco@umass.edu



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Data Dimensionality: How many variables do I have?

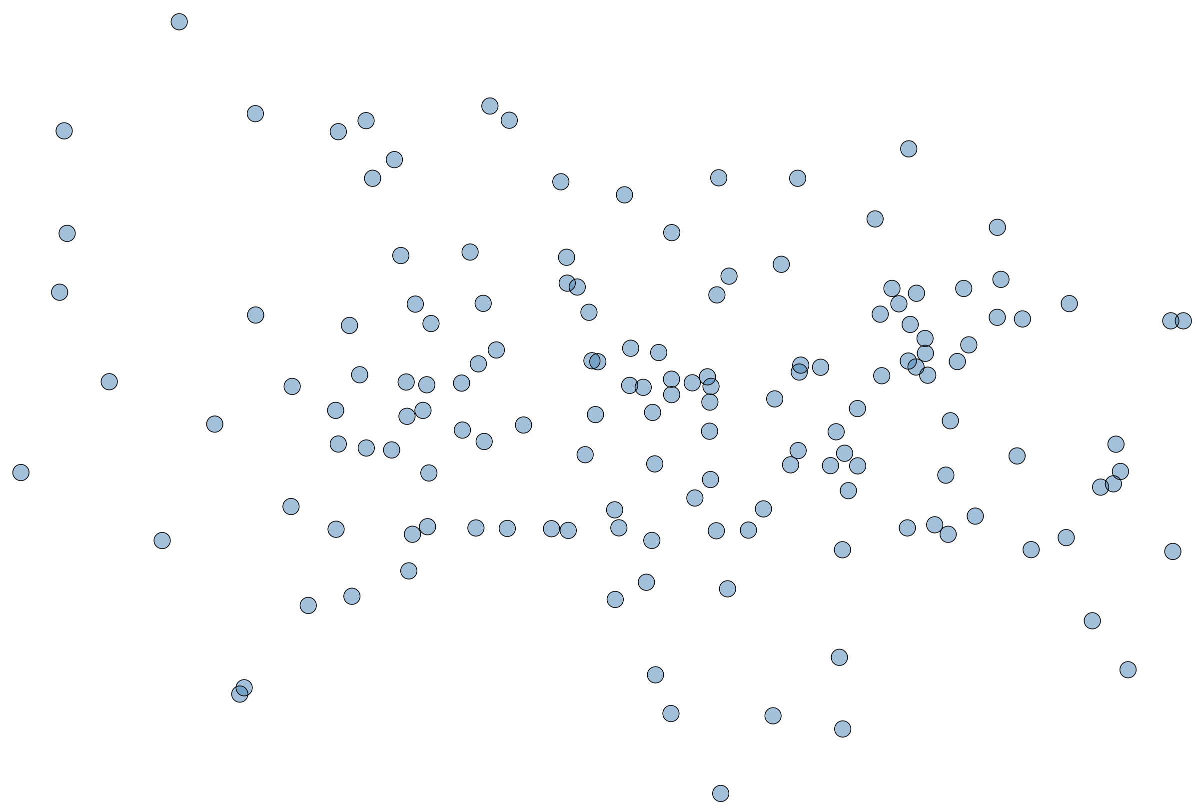
Data Dimensionality	**Visualizing data**
What is data dimensionality? • It's just the number of variables in your data. • They don't have to correspond to physical and temporal dimensions. • Axes in 'variable space' or 'parameter space'	• 1D: boxplots, histograms • 2D: conditional boxplots, scatterplots • 3D: coplots, 3D plots, 'slices' • 4D is difficult or impossible • Multiple 3D 'panels' • 5D and higher is generally impossible with single plots.

Scatterplots and Pairplots

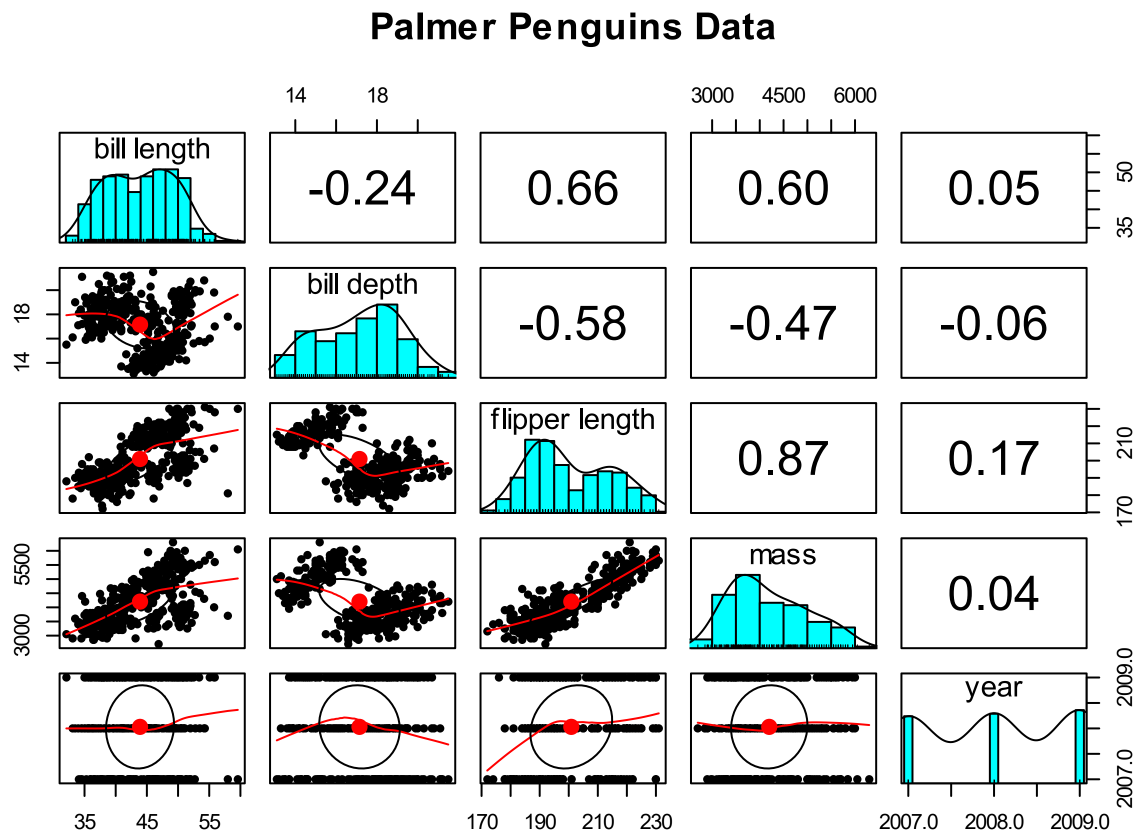
Scatterplots	Pairplots
• Good for 2-dimensional data • Requires 2 continuous variables • 3D scatterplots are possible, but they can be hard to interpret if they're not interactive.	• Matrix of 2D scatterplots. • Useful for multi-dimensional data.

Scatterplots and Pairplots

Scatterplots are useful



Pairplots are even better!



Visualizing 3D Data: Coplots

Visualize 3-Dimensional data with 2D slices

- Individual data points are plotted on x-y plane
- The z-axis is divided into bins
- Straightforward for categories
- Binning algorithm needed for continuous
- Each z-bin is flattened and plotted as 2D

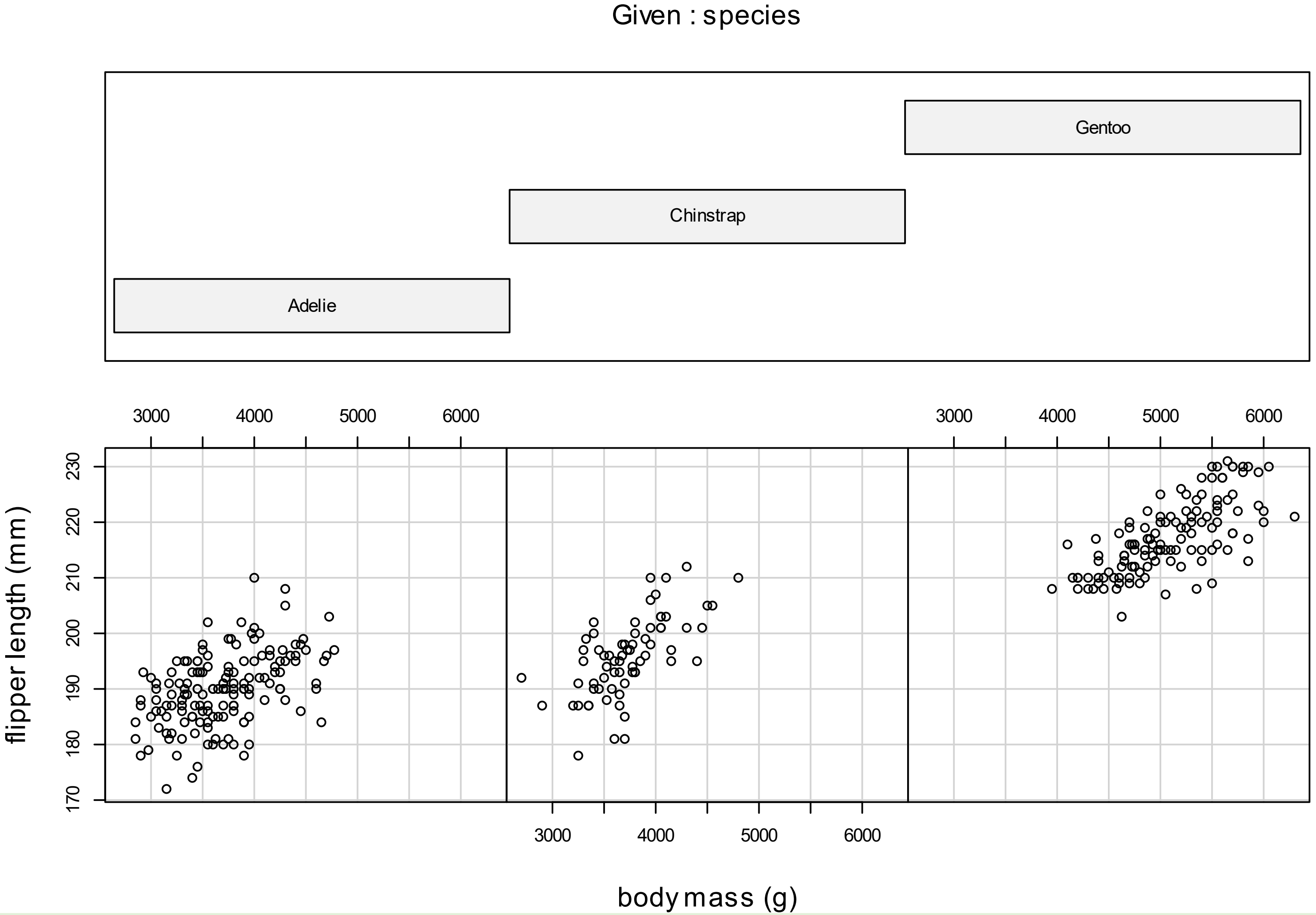
Visualizing 3D Data: Coplots

Coplot with a categorical variable

Each slice is a penguin species:

- **Adelie**
- **Chinstrap**
- **Gentoo**

What can you see?



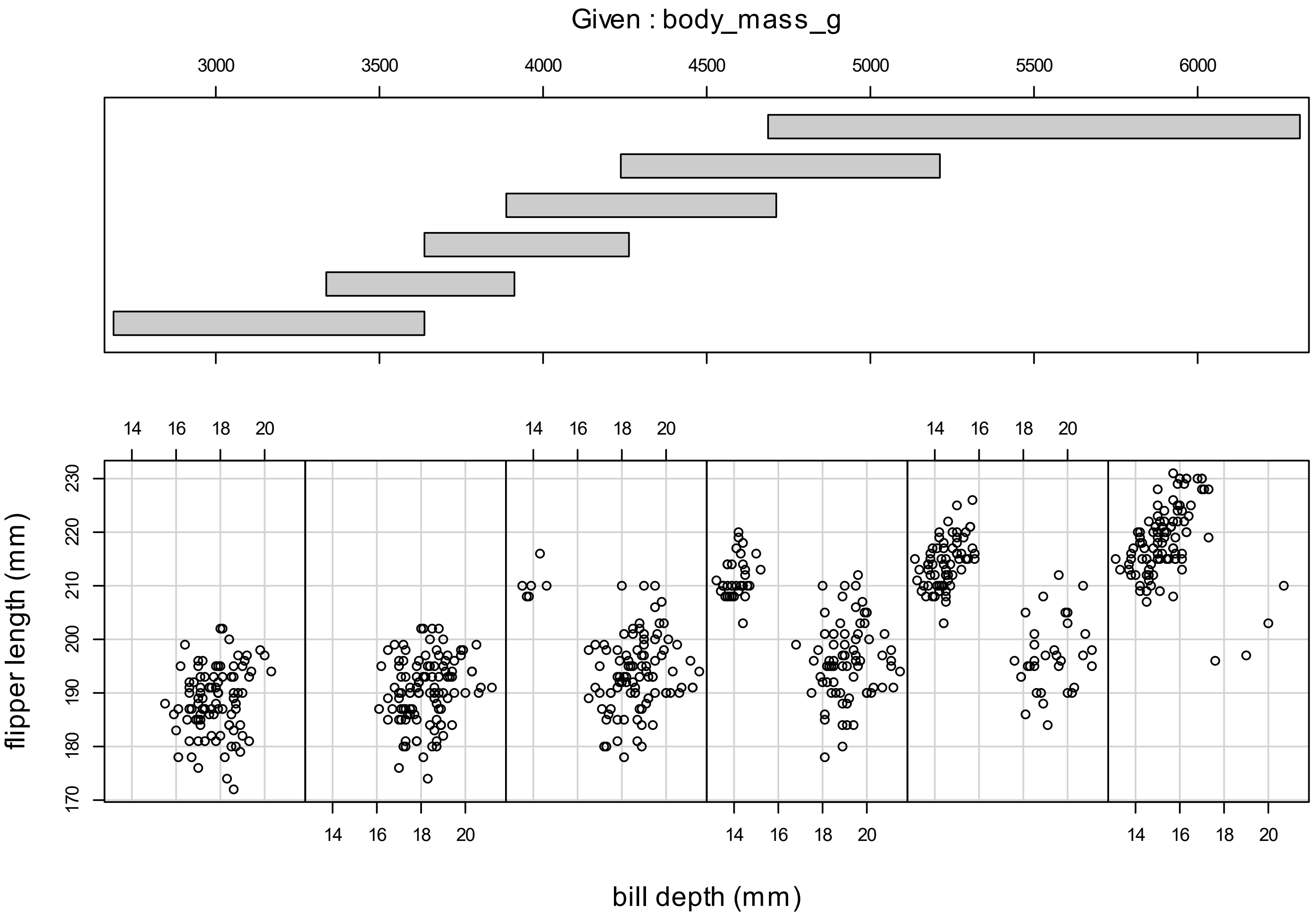
Visualizing 3D Data: Coplots

Coplot with a numeric variable

Body mass broken into 6 'bins'.

What insight does this plot show?

Can you explain the two clusters at greater body mass?



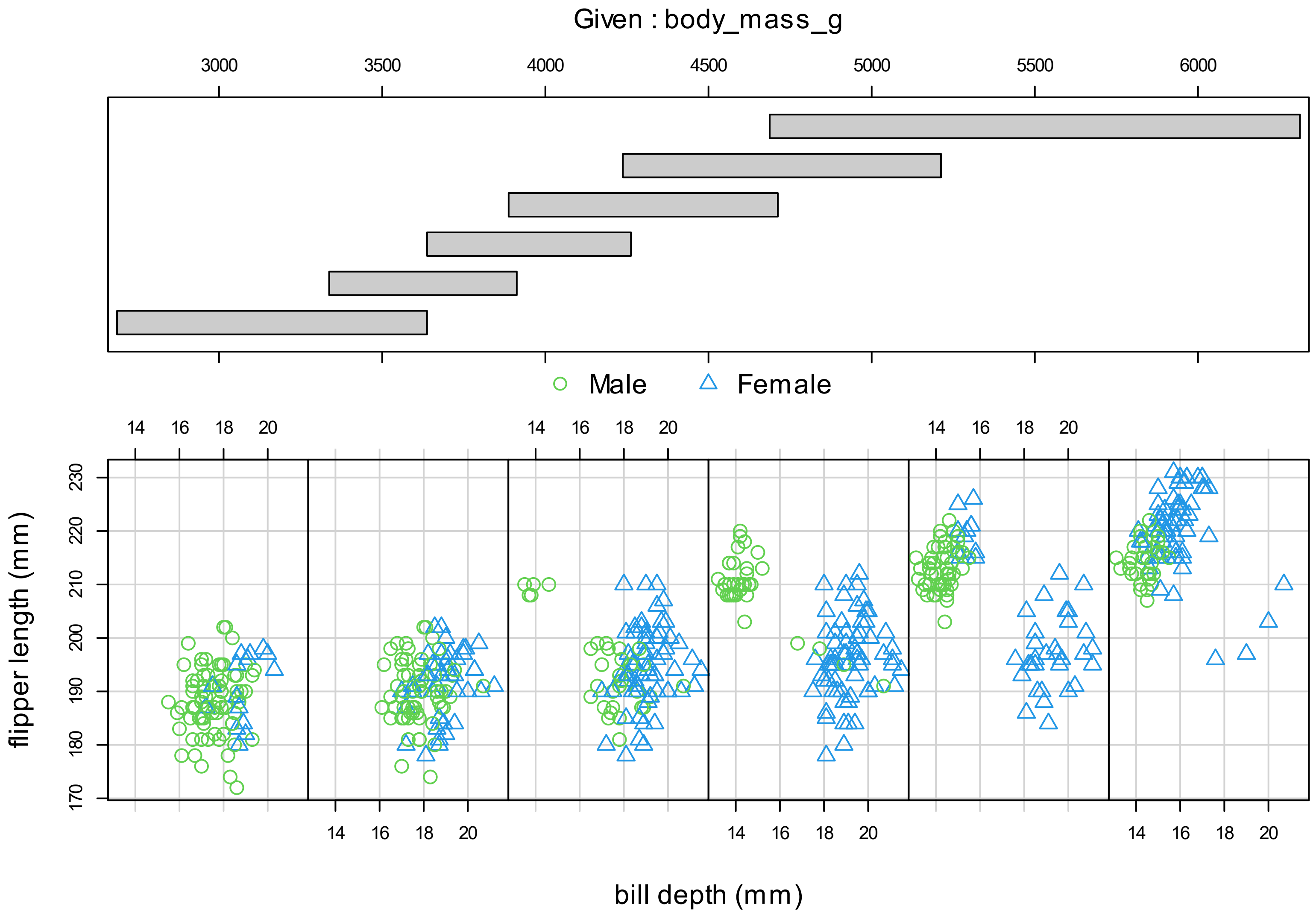
4D Slice Plots: Point Color and Shape

Coplot with a numeric conditioning variable and 4th dimension as point shape.

- 6 body mass bins
- Sex as plotting character

Do the groups make more sense now?

- What factor(s) is/are still missing?
- How could you put this into an English sentence?



3D example: Modeling Mountain Pine Beetle epidemics

3-dimensional data

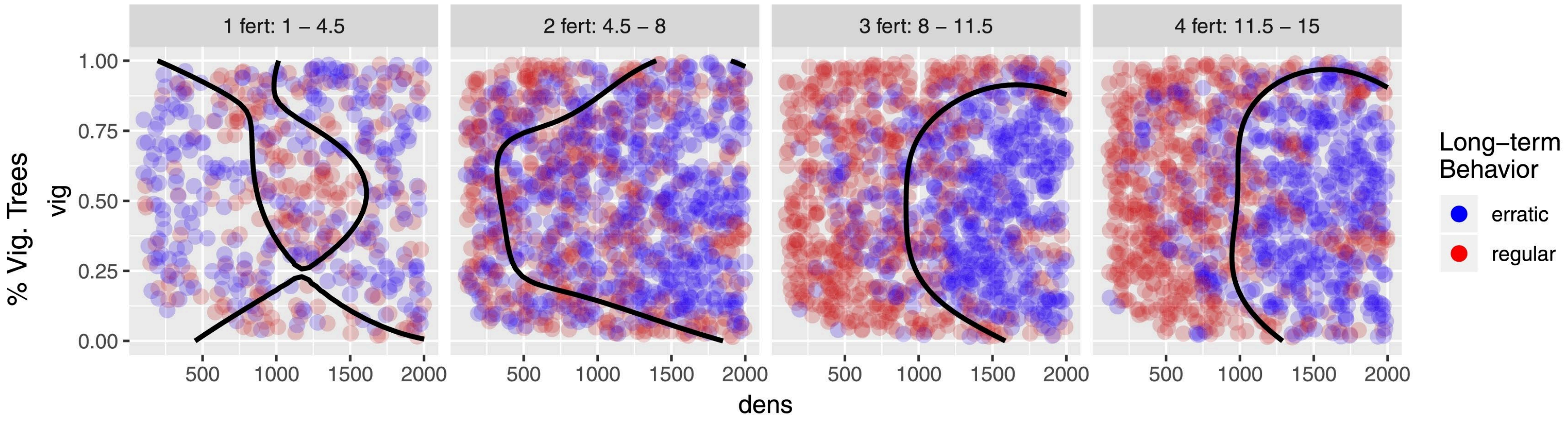
Three model parameters:

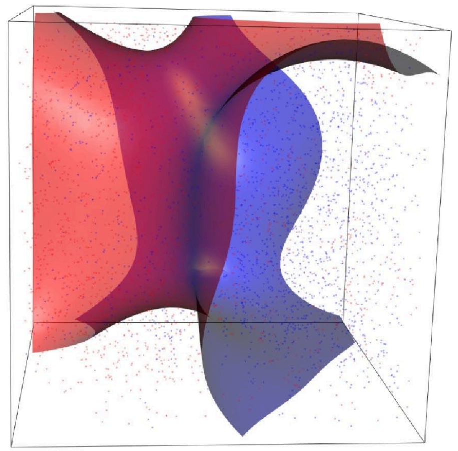
1. Beetle fertility
2. Tree vigor
3. Tree density

Two response types:

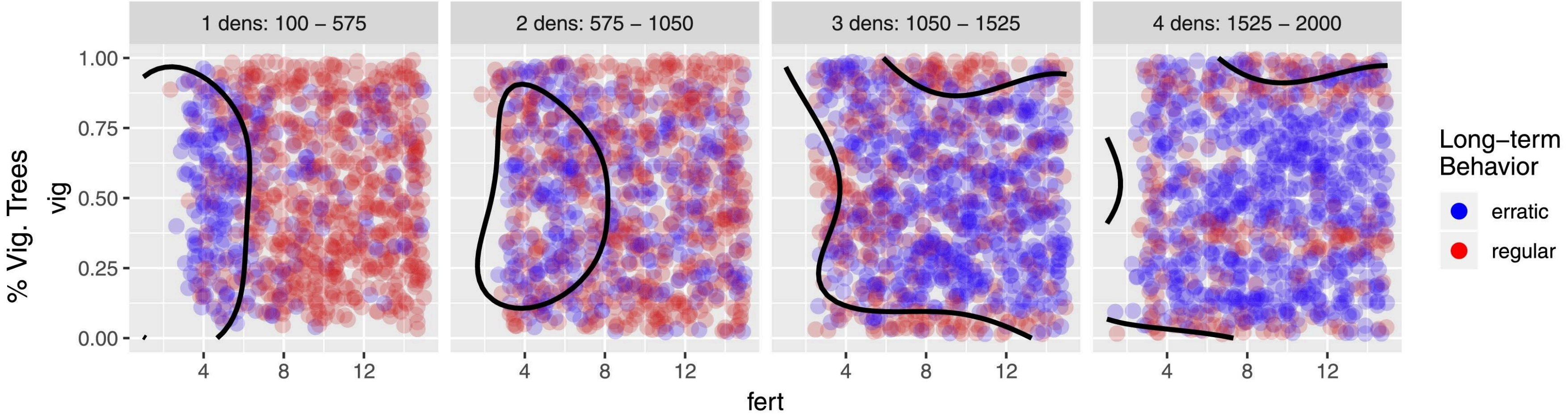
1. Long term epidemic behavior - categorical: erratic or regular epidemics
2. Epidemic proportion - continuous: long-term average percent of epidemic area

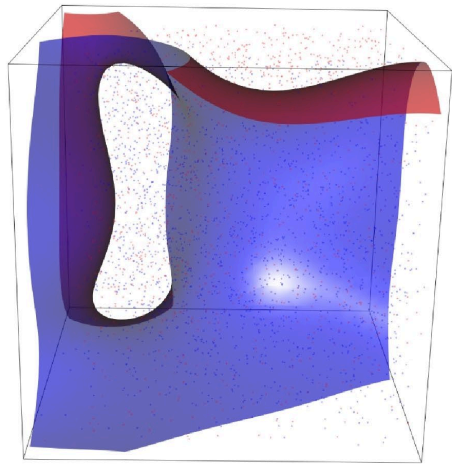
2D Slice Plots: Slices + Epidemic Behavior (discrete) Response as Color



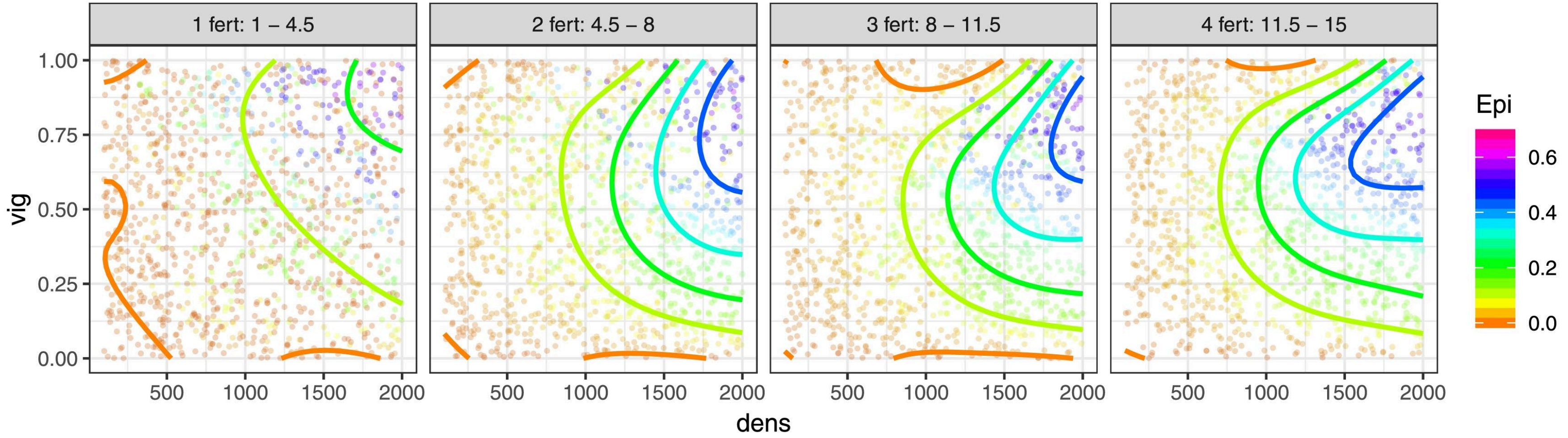


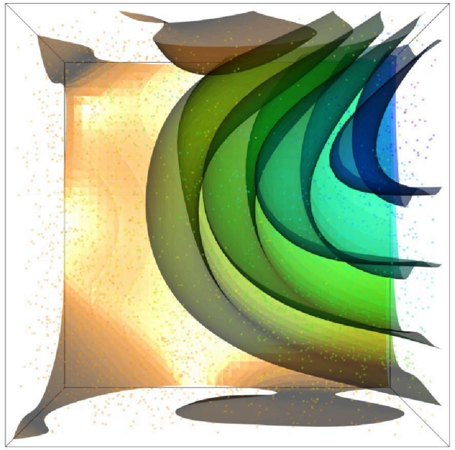
Max trees per ha.

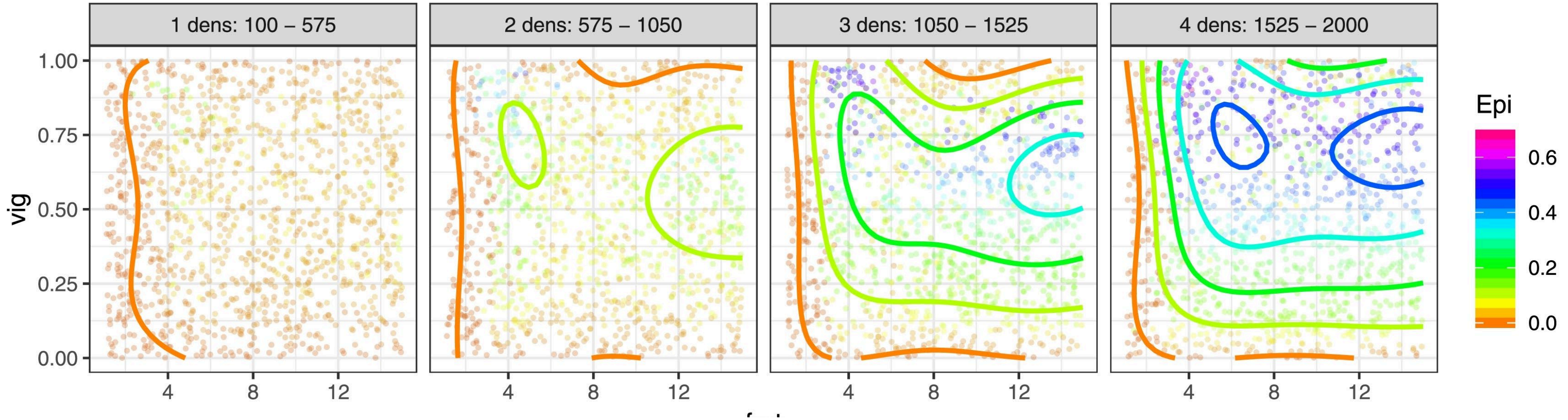


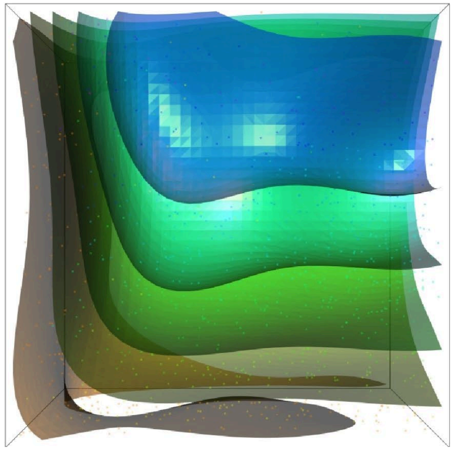


2D Slice Plots: Slices + Epidemic proportion (Continuous Response) as Color



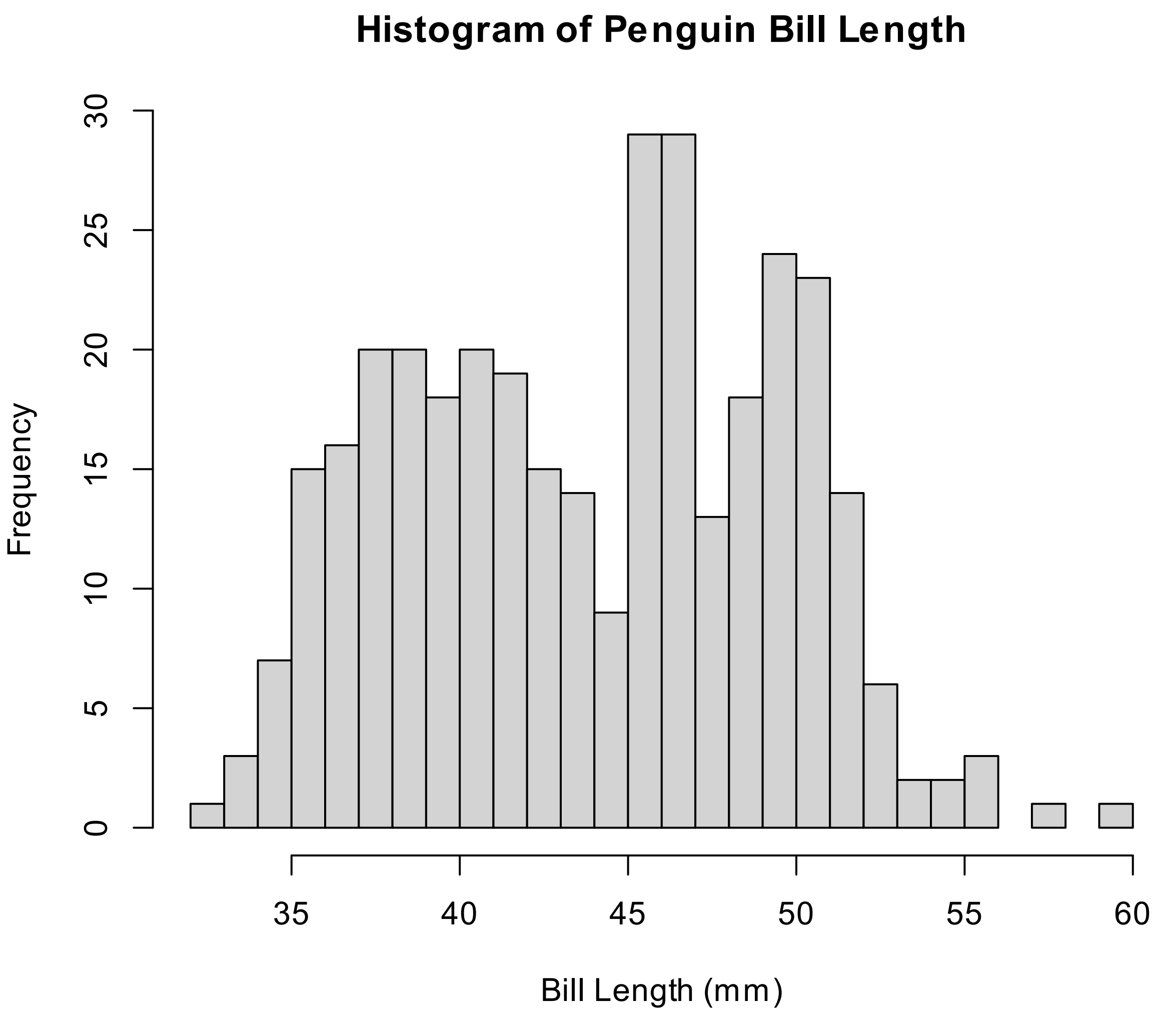






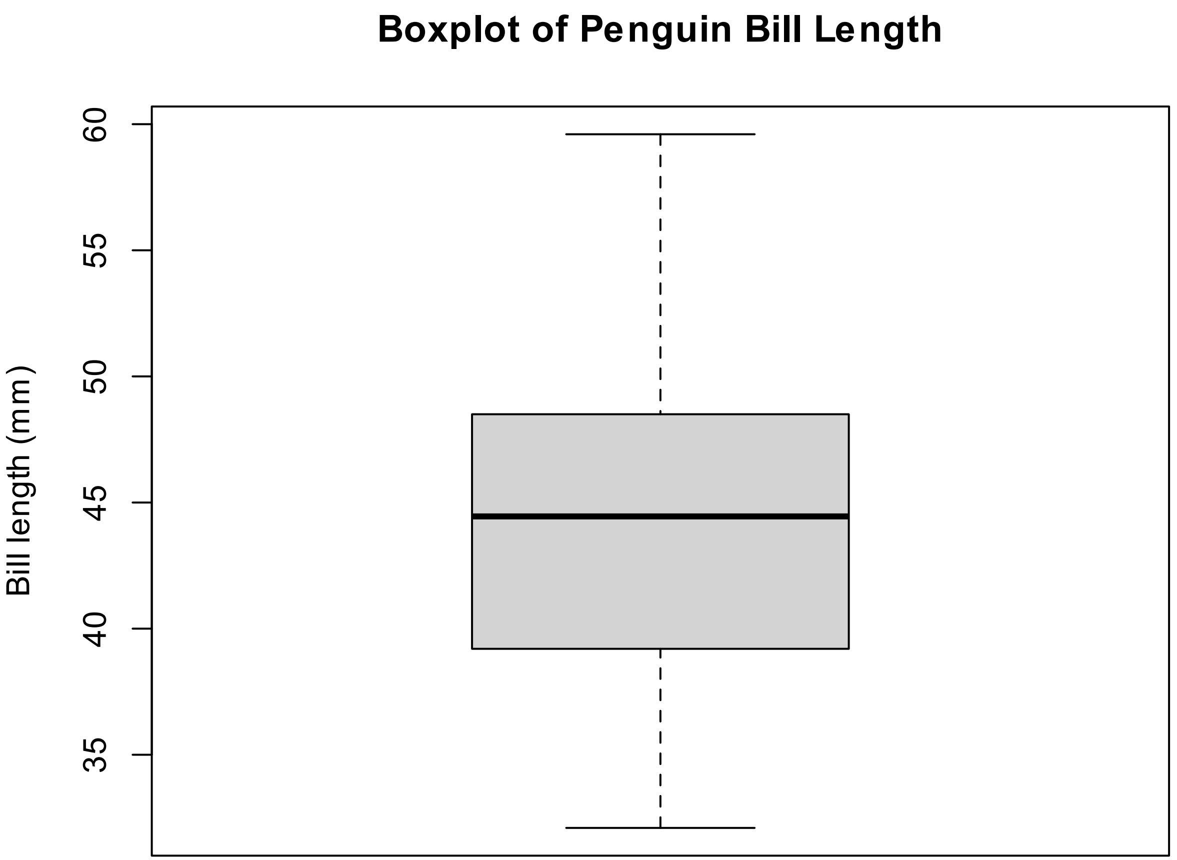
Univariate Data: Histograms

- Show the 'shape of the distribution' of the data.
- Sometimes called frequency diagrams.
- Data are aggregated into bins



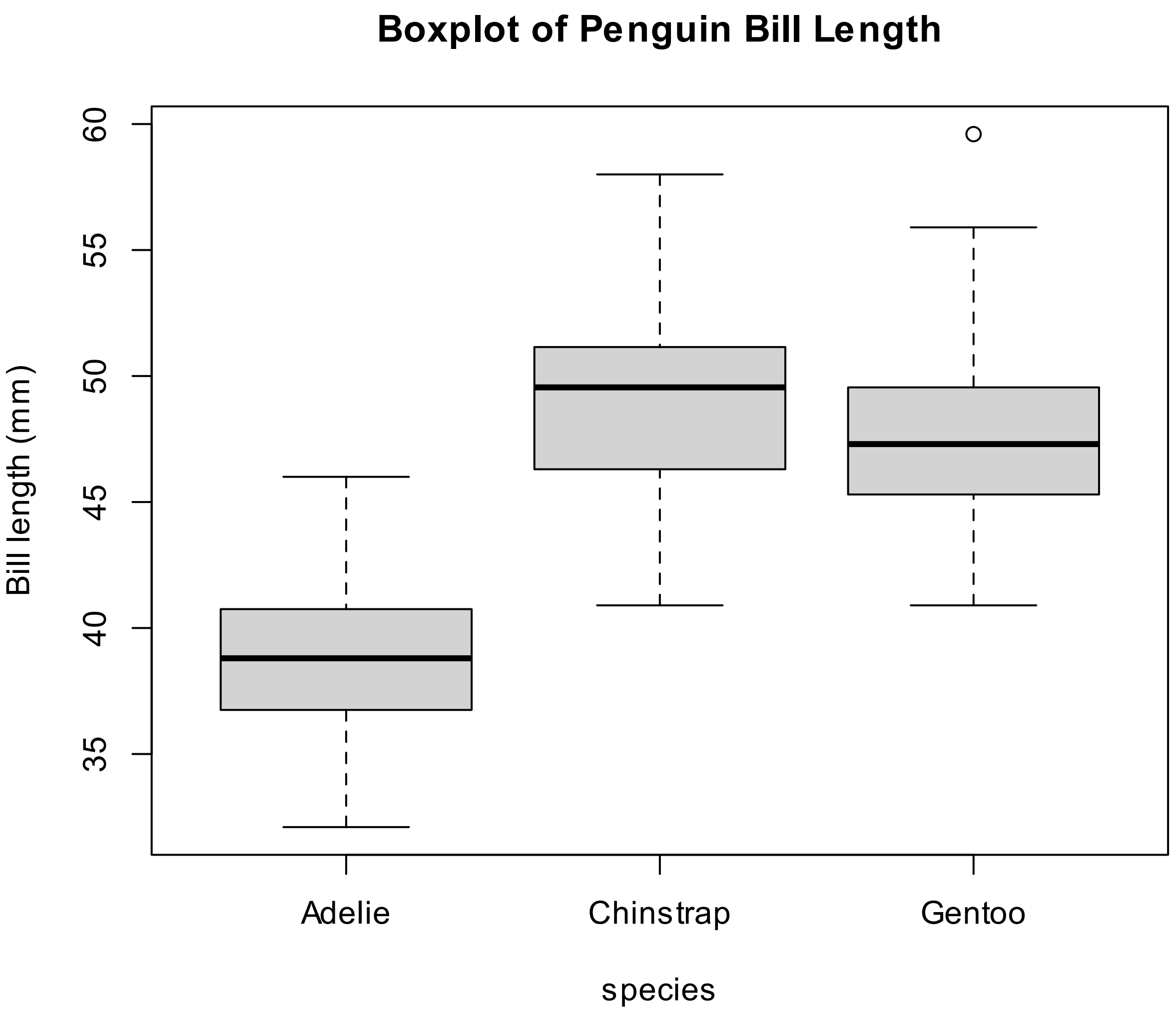
Univariate Data: Boxplots

- Summarizes data: like a 5-number summary.
  - IQR, Median
  - Whiskers: it's complicated
    - https://www.r-bloggers.com/2012/06/whisker-of-boxplot/
- Univariate: but can be made bivariate, or even 3D with grouping factors.
- Gives you an idea about spread and skew.
  - How is it different from a histogram?



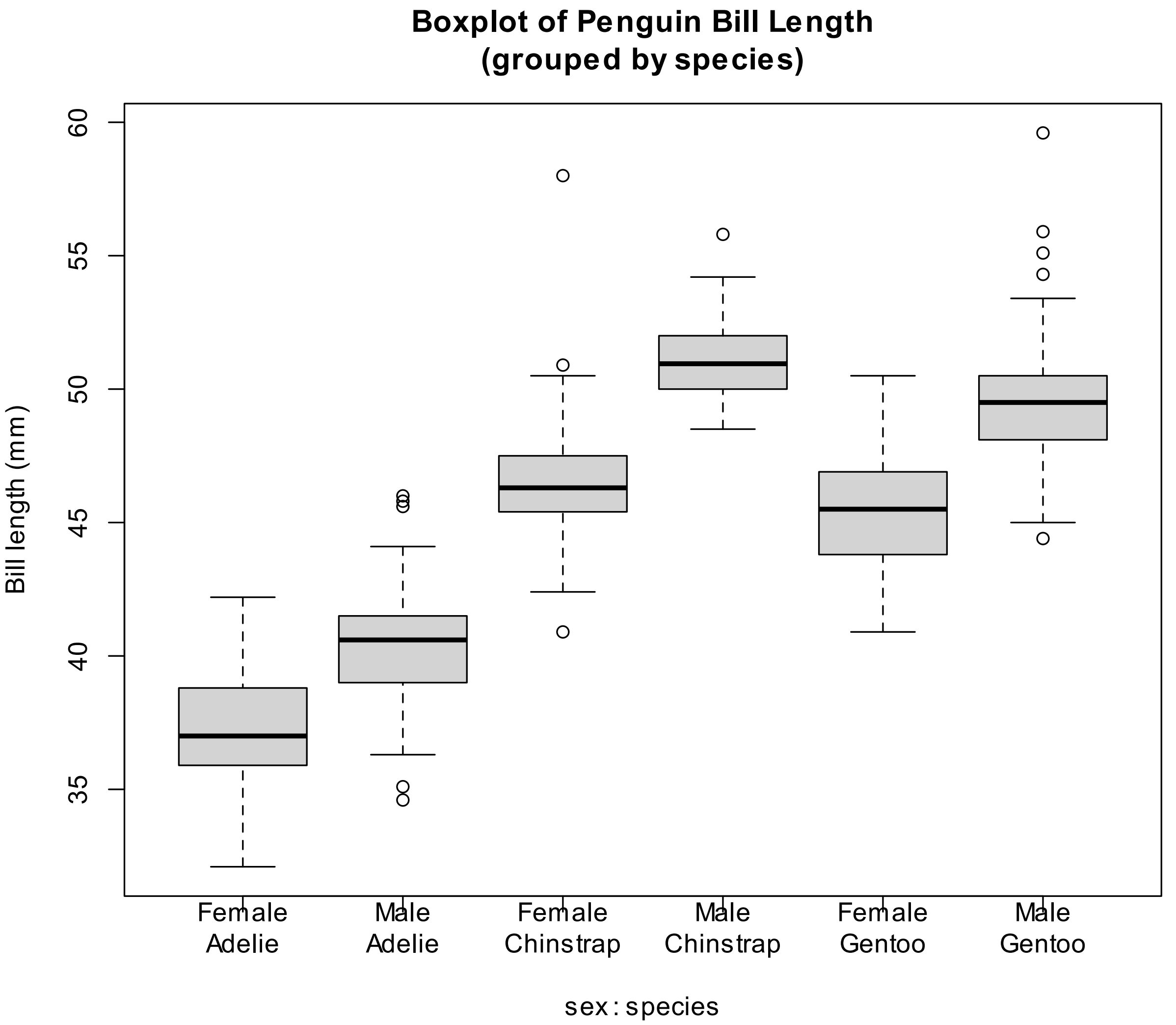
Conditional Boxplot: Univariate with Grouping Factor

- Univariate Data, aggregated by a grouping factor.
  - Kind of like a coplot
- Summarizes data just like a regular boxplot



Doubly Conditional Boxplot

- Two grouping factors
- 3D data



Graphical Exploration Recap

- Associations
- Tools to describe associations
  - Spearman and Pearson correlation
  - Graphical exploration
- Data dimensionality and plotting
  - Bivariate and univariate plots
- Plots that show all data: coplots, scatterplots, 3D scatterplots, others?
- Plots that show aggregated data: histograms, boxplots, others?
- Grouping factors (conditioning variables)

In-Class Data Exploration

- NOTE: Moodle group self-select will be slightly different going forward!
  - Setting was wrong on last Thursday's assignment
  - Use the group self-select for every assignment going forward.
  - Group names will have to be slightly different each time (it's a pain, but it's the best I can do to keep track of groups…)
  - Feel free to shuffle groups as you wish, that's why I have a different group/grouping set up each time.

Announcements

- Re-load the main course page often!
- Remember office hours: great time to ask questions.
  - Tuesday/Thursday 1-2
  - By appointment (email me!)
- Echo360 Lecture recordings.
- Several assignments are due on the 25th:
  - Check the 'Upcoming events' feature on Moodle to see the timing of due dates.

COLLEGE DAY COOKOUT

Time: 11 a.m. – 1 p.m.

Date: Friday, September 23

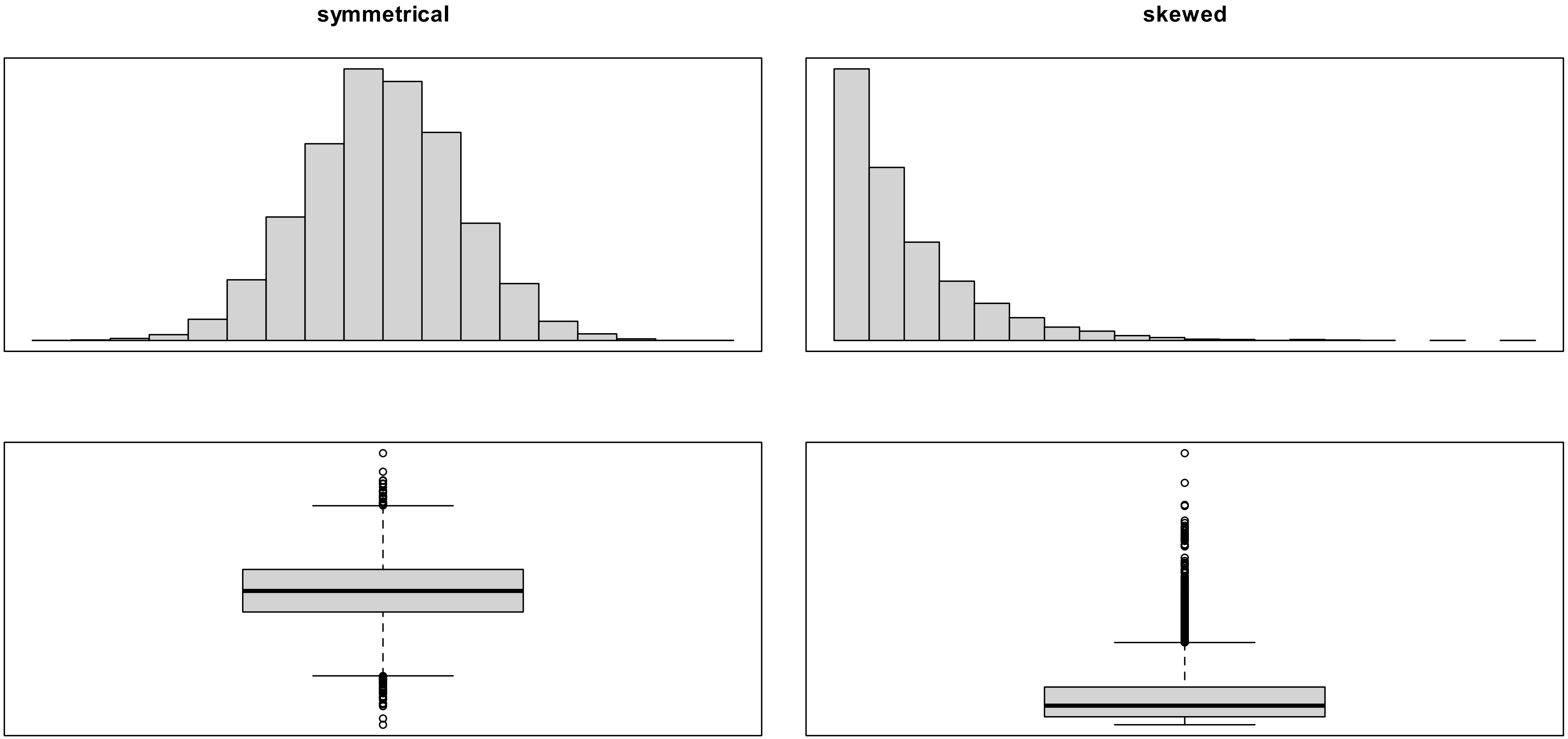
Location: Integrated Science Building (ISB) Courtyard!

EAT GREAT FOOD!

MEET NEW FRIENDS!

UMass Amherst | College of Natural Sciences

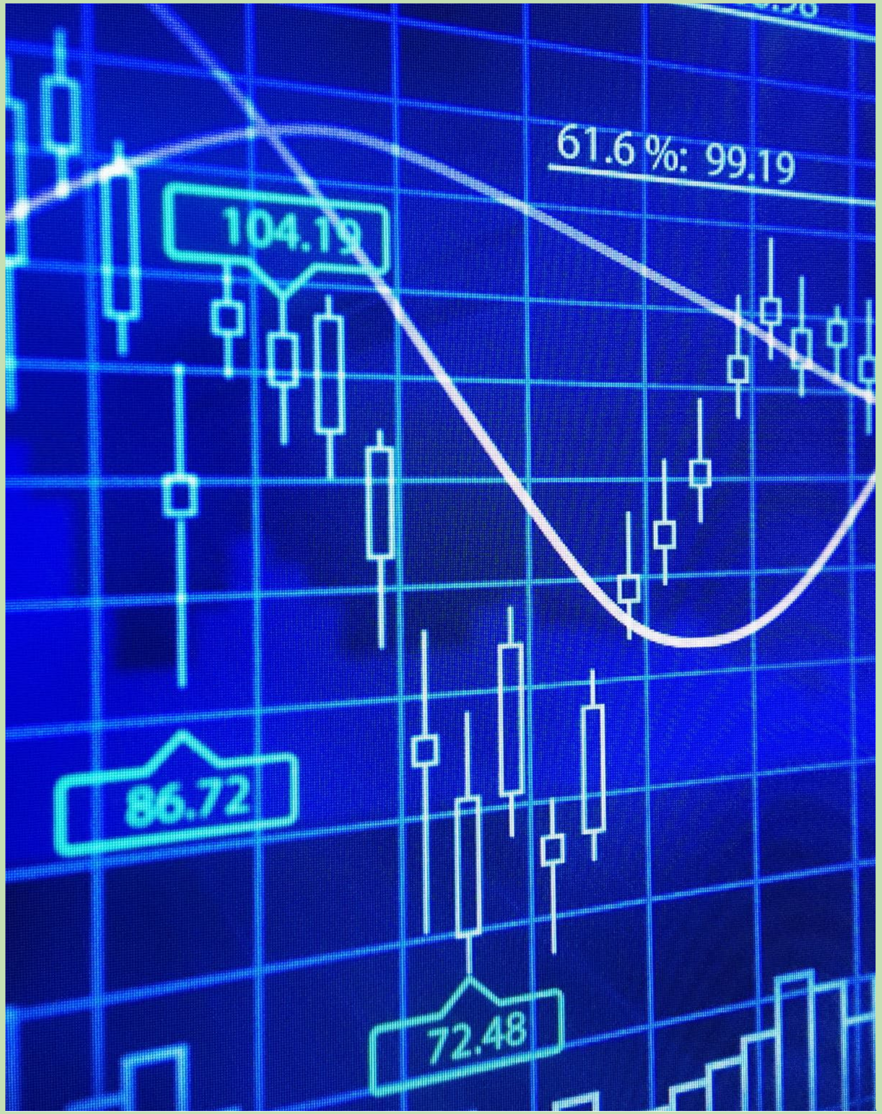
Symmetrical and Skewed Data: Histogram and Boxplot



Function basics: important function terms

- **monotonic**, **asymptotic**, and **divergent**
- **variables** and **constants**
- **powers** and **exponents**
- **local linearity**
- **domains: bounded and unbounded**
- **sums** and integrals
- continuity, slope, and step functions
- saturating, diminishing returns
- inverses

variables and constants



- Constants may also be referred to as parameters

How many variables are there in this equation?

$$P(x) = \frac{1}{\sigma\ \sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

- A good strategy to simplify the form of a function is to set all constants to zero or one. That way you can eliminate them from the formula, leaving only the variables.

- Hint: How many times does *x* occur?

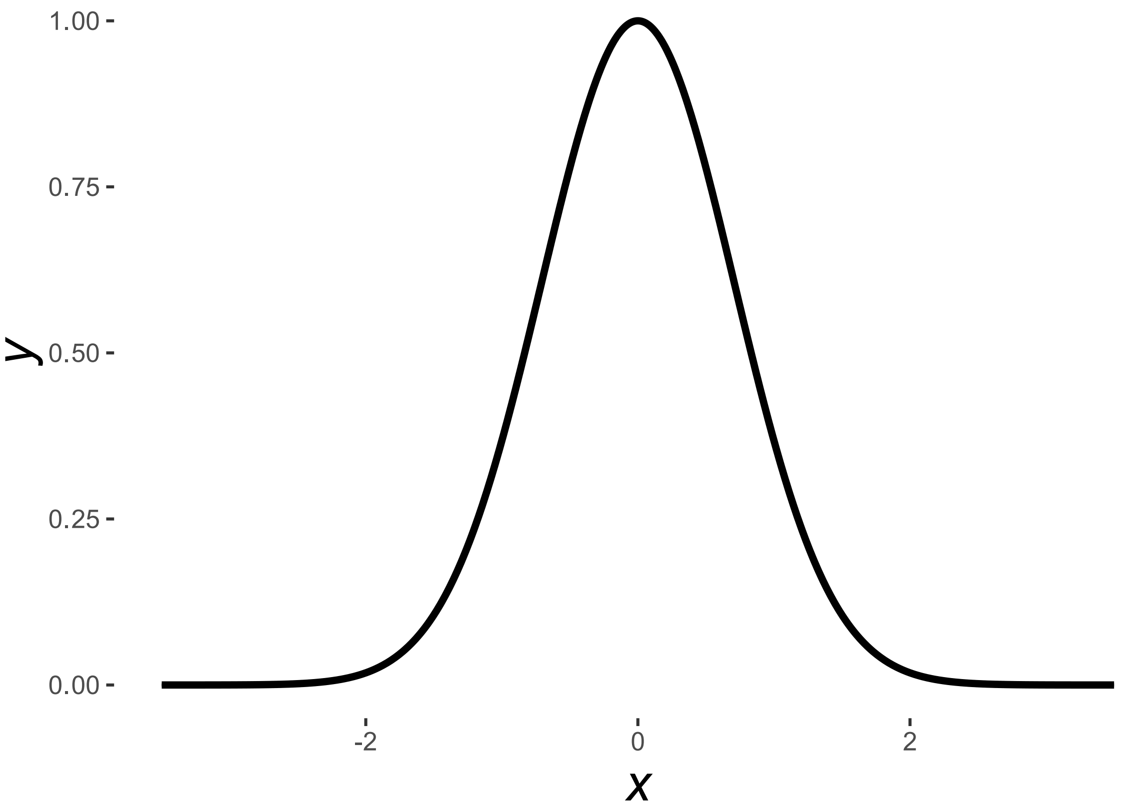
Variables and constants, i.e. parameters

How many variables are there in this equation?	**That is considerably simpler to understand than the original monstrosity!**
• Only 1: x • Use the strategy above to eliminate the constants: $$P(x) = e^{-x^2}$$	$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

Variables and constants, i.e. parameters

We can plot this: it looks like the Normal curve!

$$P(x) = e^{-x^2}$$



Class of functions

Common functions we use as deterministic models:

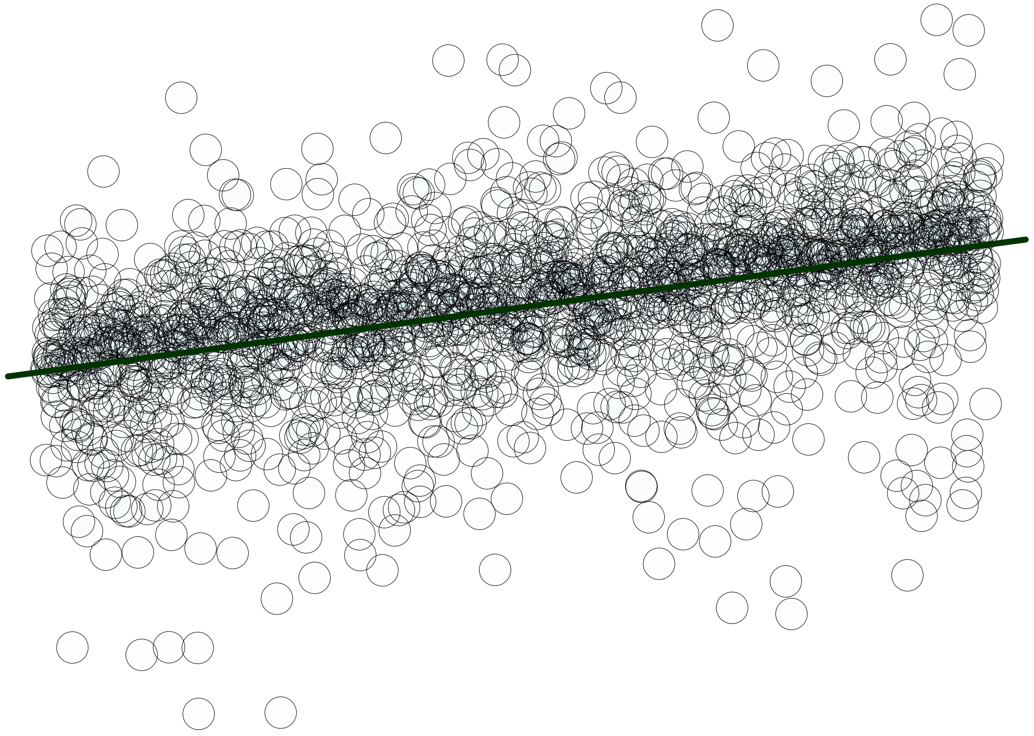
- linear
- polynomial, rational, and power
- exponential (and logarithmic)
- periodic
- combination functions

Linear functions

Linear functions have variables raised to a power of 1.

- Can be one or more variable:
  - $y = mx + b$
  - $y = m_1 x_1 + m_2 x_2 + \ldots + m_n x_n + b$
- Statistical literature likes to use alphas, and betas
  - $y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \ldots + \beta_n x_{ni} + \epsilon$
  - $y_i = \alpha + \beta_1 x_{1i} + \beta_2 x_{2i} + \ldots + \beta_n x_{ni} + \epsilon$
- Key features:
  - The *variables*, i.e. the x_i ,are first-degree.
  - Each *variable* is multiplied by a *parameter*, the β_i is multiplied by each value of x

A linear model is always a great place to start.



Linear models: interpretation

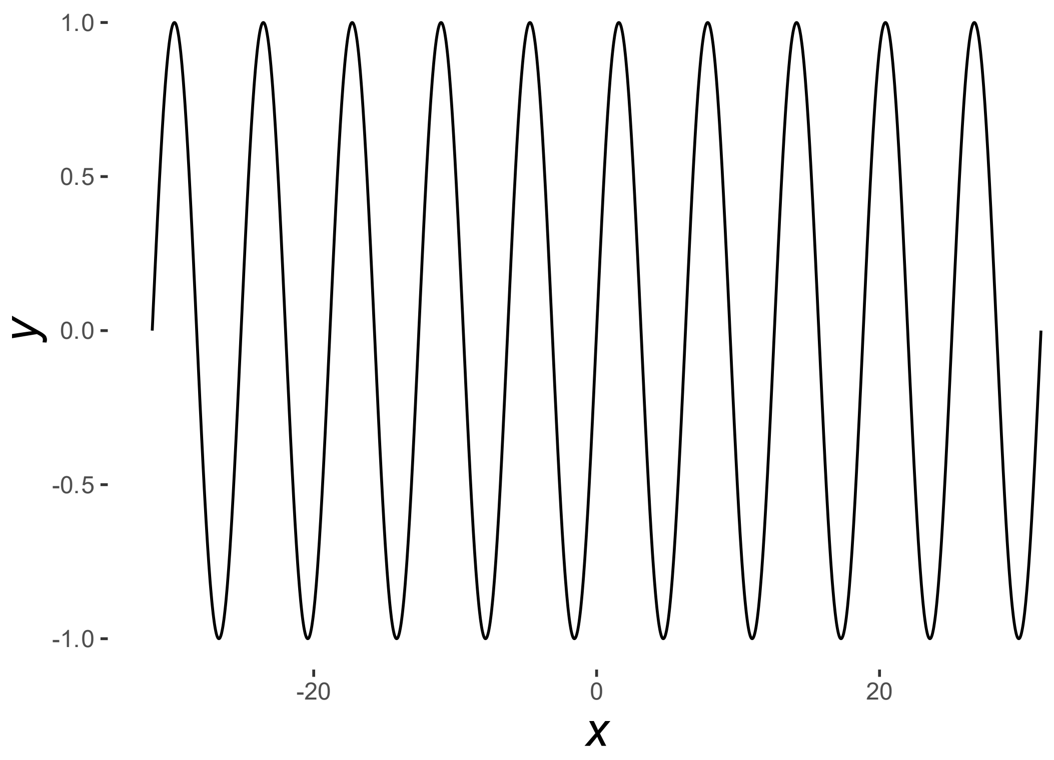
A linear model describes a *constant rate of change*

- Reed canary grass plant biomass increases by 0.71 grams for each additional gram of added soil nitrogen per cubic meter.
  - The rate of increase is constant everywhere:
  - If soil with 1g nitrogen results in biomass of 1 g.
    - Soil with 2g nitrogen: expected biomass: 1.71 g.
  - If soil with 3000g nitrogen results in biomass of 100 g.
    - Soil with 3001g nitrogen: expected biomass: 100.71 g.

Is the constant rate of change reasonable?

Local linearity

Could you model this curve with a linear function?



Local linearity

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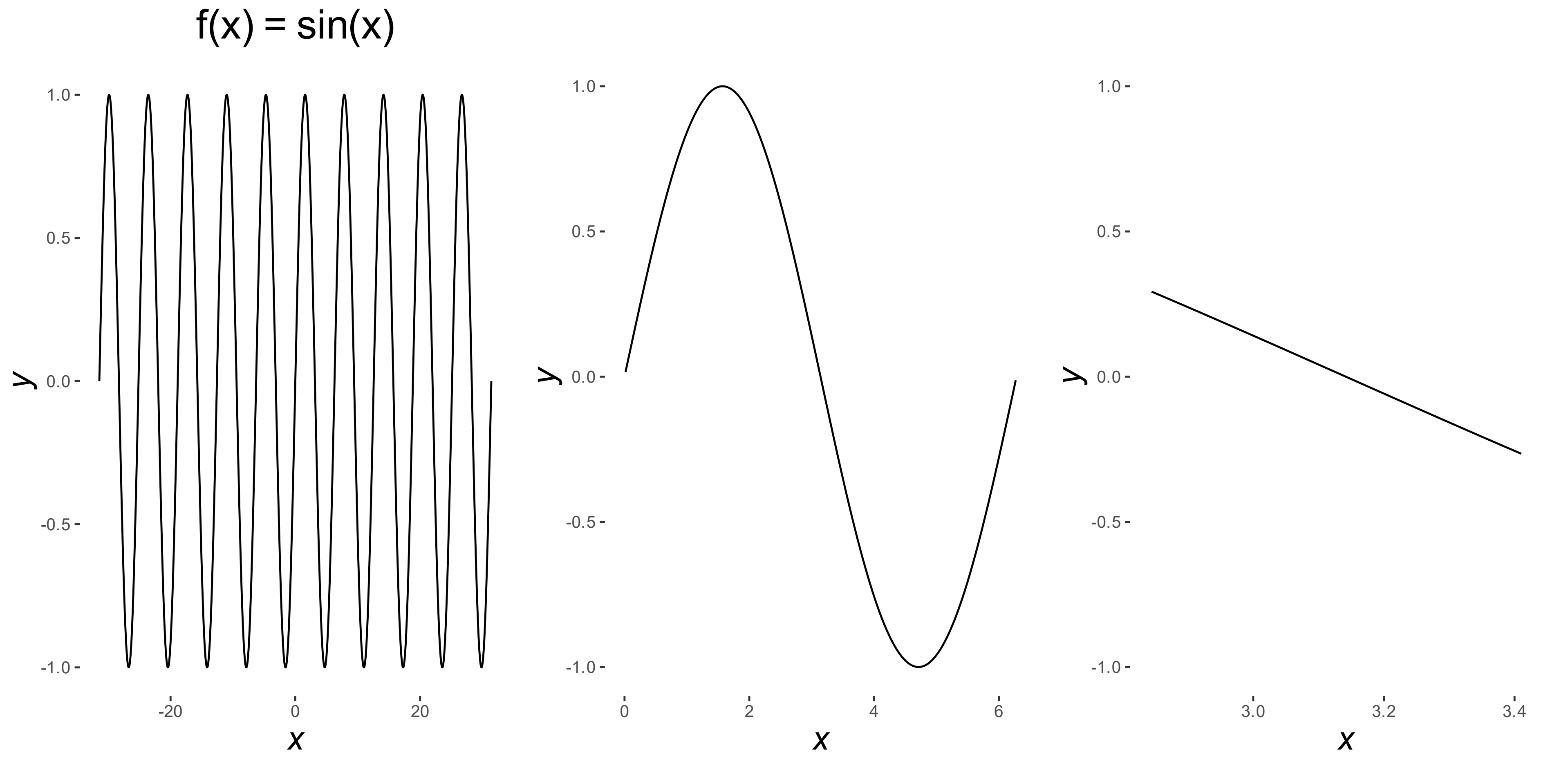


Local linearity

We are often justified in using a linear model, even when we know a relationship isn’t linear:

- If we are only interested in a small subset of the range of predictor values.
- All (most) continuous functions look very linear if you zoom in.
- Linear functions are much simpler than the rest of the functions we’ll consider.

Atlas of Function Classes

A selection

Notation for Polynomial Functions

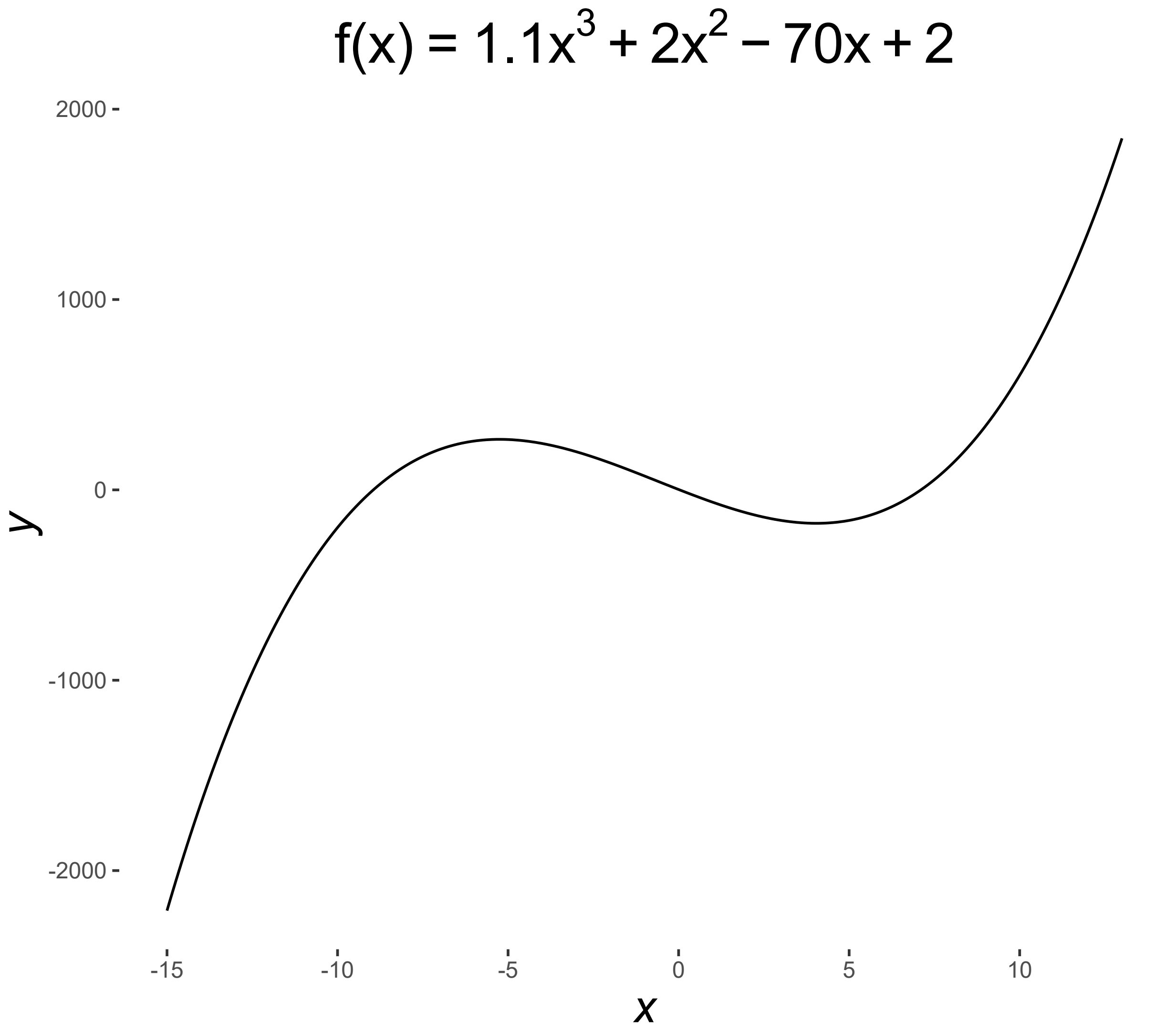
What are bases and exponents?	Notation convention:
Let a be a real number, and b be an integer: a is the base b is the exponent Exponentiation in this world means: "Multiply `a` by itself `b` times."	a^b

Polynomial functions

Polynomial functions have non-negative integer powers:

$$f(x) = x \; f(x) = x^3 - 2 \times x^2$$

Linear functions are a subset of polynomial functions.



Polynomial models

Polynomial terms are sometimes added to models to improve the **model fit**.

- Polynomial models are typically *phenomenological*.
- There's usually not a clear biological or ecological interpretation.
- You can think of them as *tuning* parameters to increase model fit, or to help with normality of the residuals.

$$y_i = \alpha + \beta_1 x_1 + \beta_2 x_1^2 + \beta_3 x_2$$

- Notice this polynomial model is *linear* in the *parameters*!

What does 'linear in the parameters' mean?

Polynomial models

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- Notice this polynomial model is *linear* in the *parameters*!

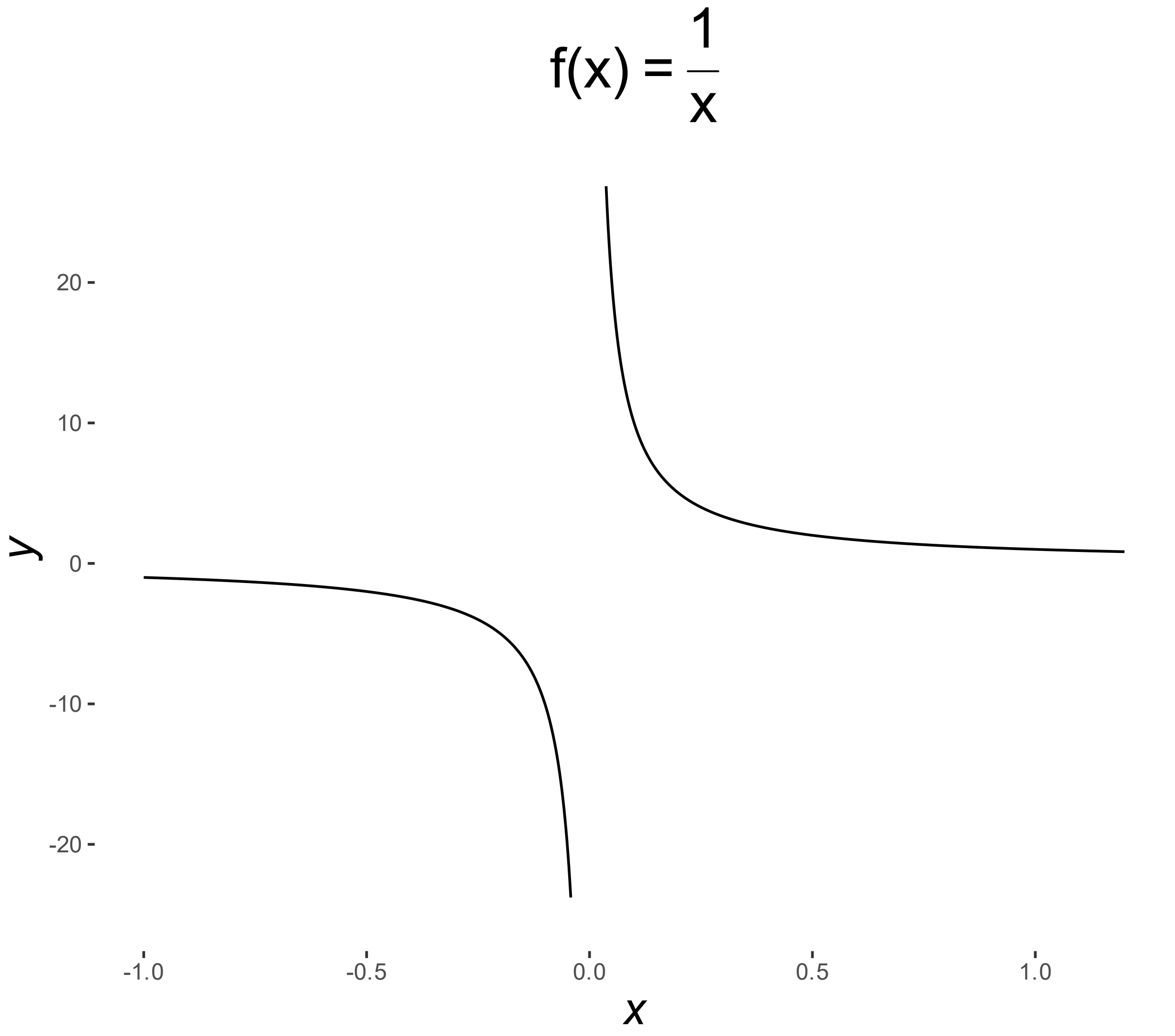
What does 'linear in the parameters' mean?

- The parameters (the betas) are not bases or exponents.

Rational functions

Rational functions can be expressed as a ratio of *polynomial* functions.

- Polynomial functions are a subset of rational functions.
- Rational functions can be *discontinuous*: division by zero.

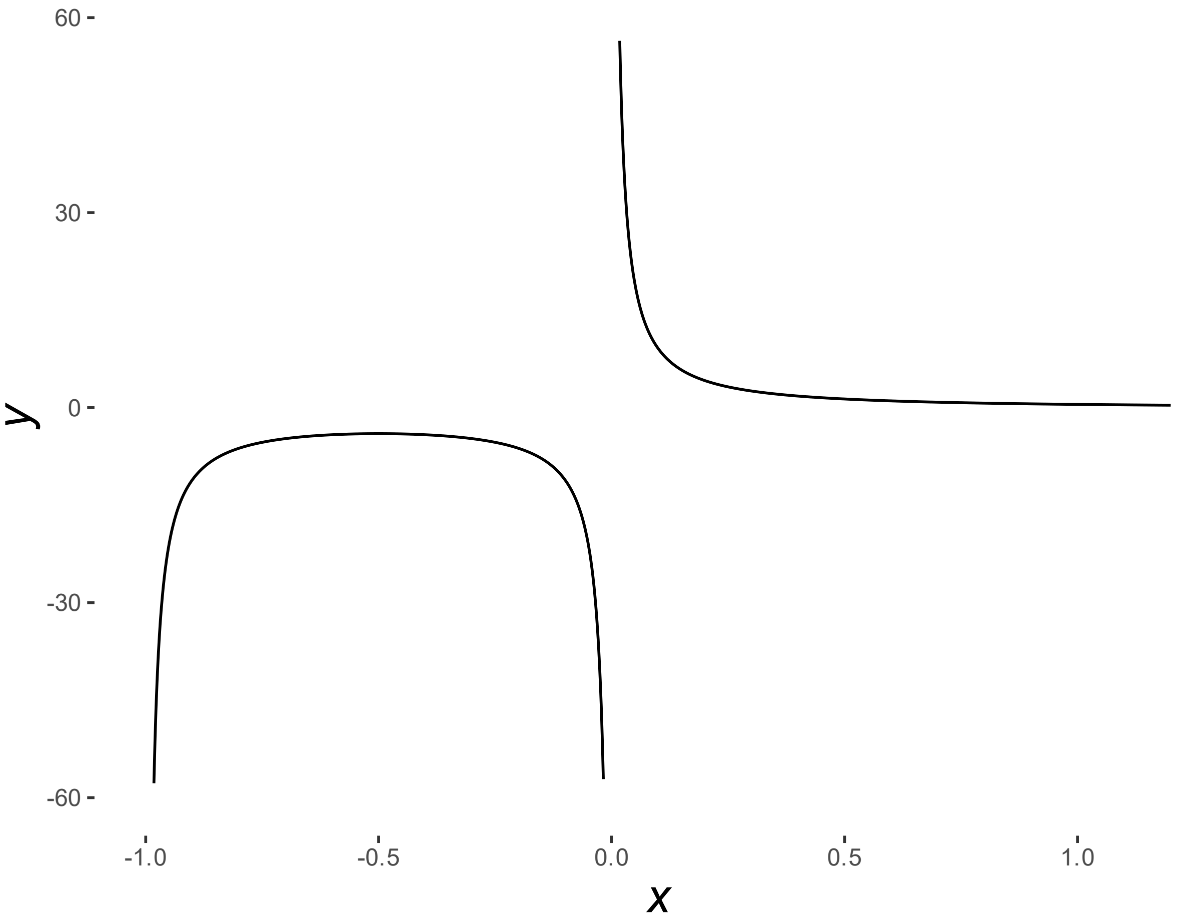


Rational functions

Rational functions are typically used in phenomenological models.

- Rational functions can emulate very complicated curves
- Tuning, improving normality of residuals, etc.
- Not used as often as polynomial or power law/fractional exponent functions

$$f(x) = \frac{1}{x + x^2}$$



Fractional or real number exponents

Functions in which the exponents can be expressed as fractions (rational numbers).	**McGarigal calls these *power law functions*.**
The square root function ***is*** a fractional exponent: $$\sqrt{x} = x^{\frac{1}{2}}$$ Rational functions are typically used in phenomenological models. • Often a result of *tuning* procedures like the Box-Cox transformation.	• There are so-called power-law distributions (like the Pareto), but we won't be talking much about these

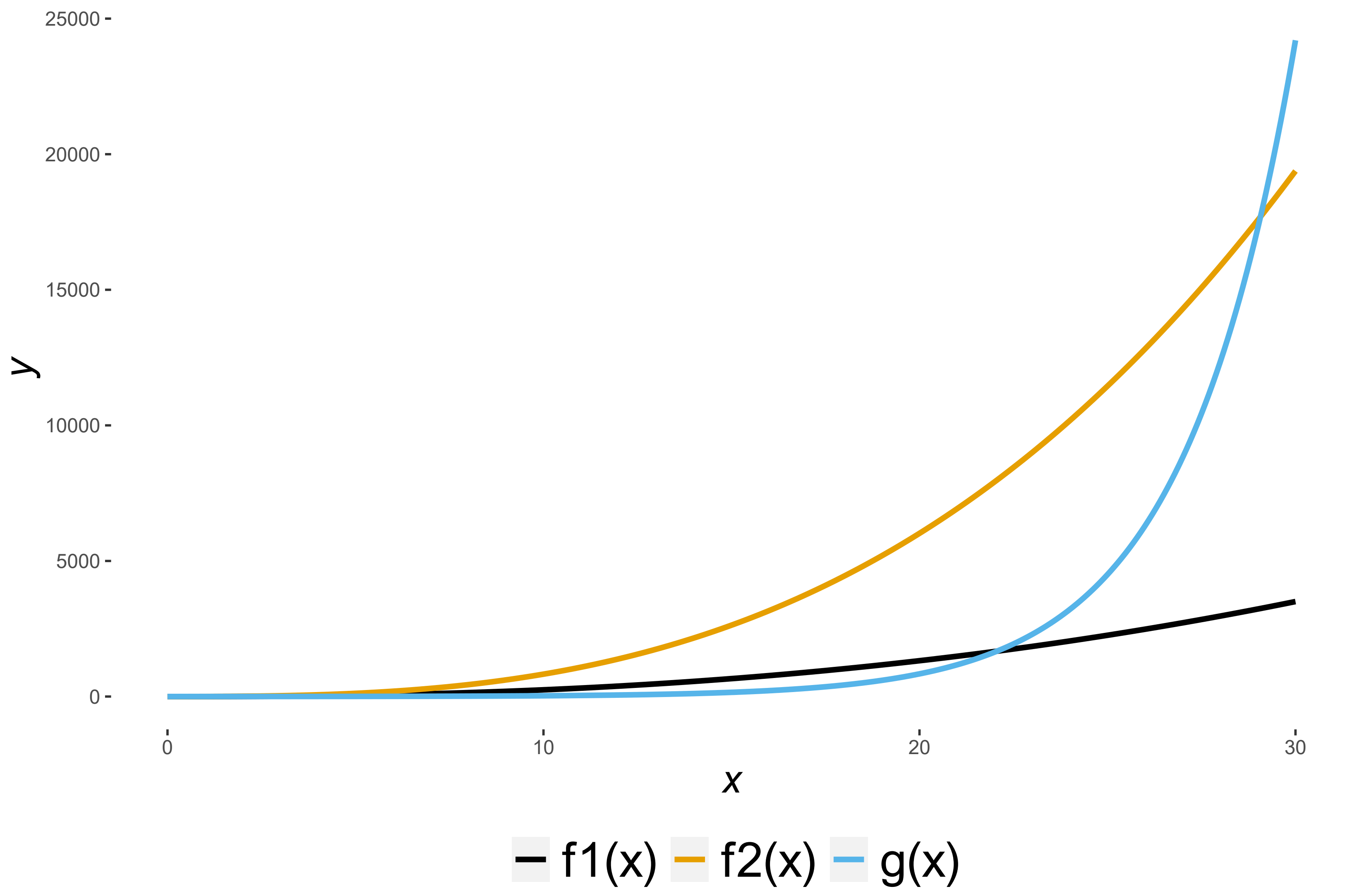
Power vs. exponential functions

Which of these functions grows fastest?

$$f1(x) = x^{2.4} - x^{0.5}$$
$$f2(x) = x^{2.9} + x^{1.5}$$
$$g(x) = 1.4^x - 0.1^x$$

- Exponential will always* win, eventually

*subject to terms and conditions



Power vs. exponetial functions

In the **long term** exponentials always grow faster than any power (i.e. rational) function.

- A rational function may grow faster initially, but an exponential term always wins as x approaches infinity.
- An exponential beats any power. But the *gamma* function wins against an exponential…

Powers: the variable is the base; the power is a constant.

$$f(x) = x^{2.4} - x^{0.5}$$

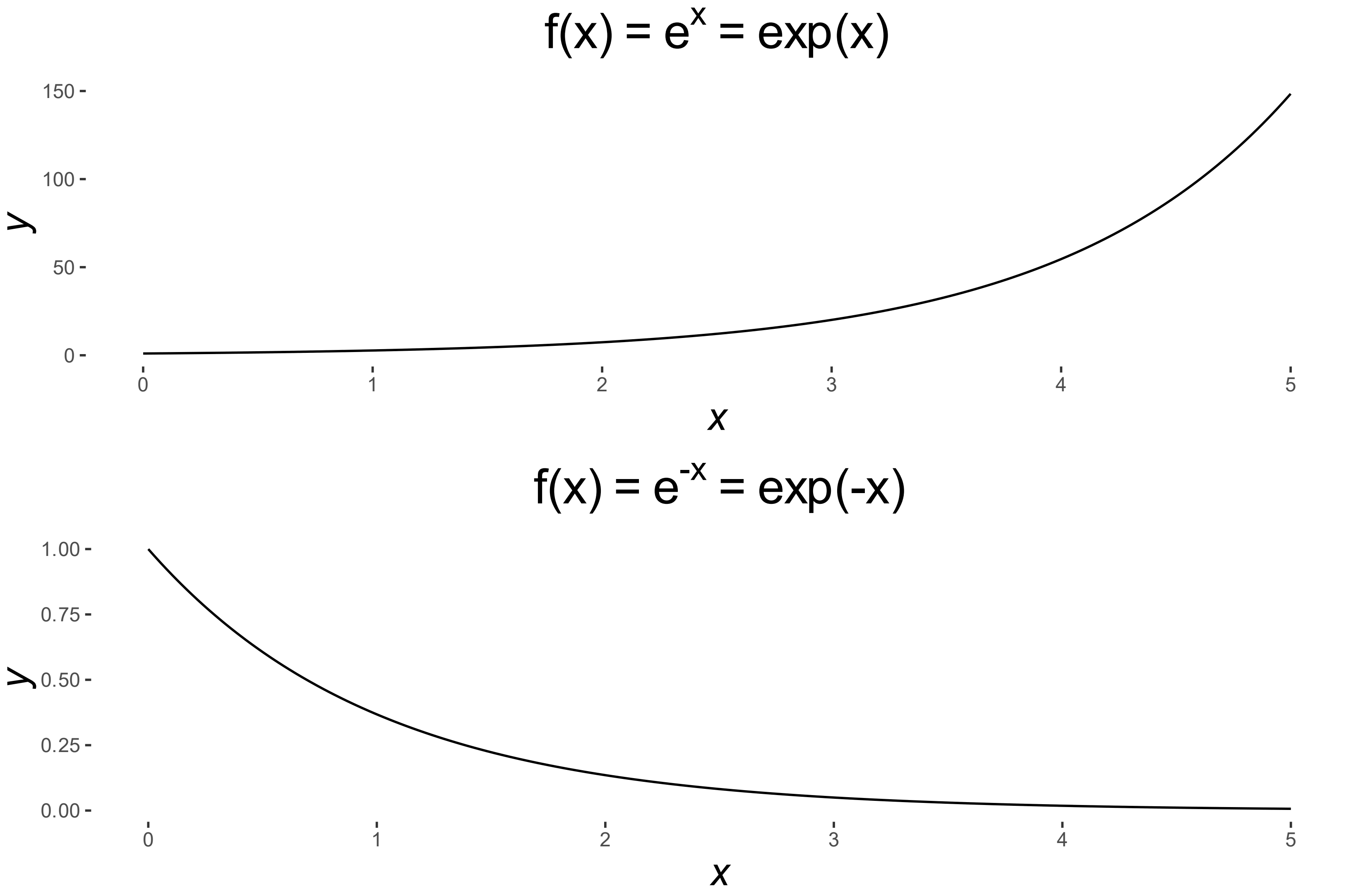
Expontials: the variable is the exponent; the base is a constant.

$$g(x) = 2.4^{x} - 0.5^{x}$$

Exponential functions

Exponential functions have the variable as the exponent.

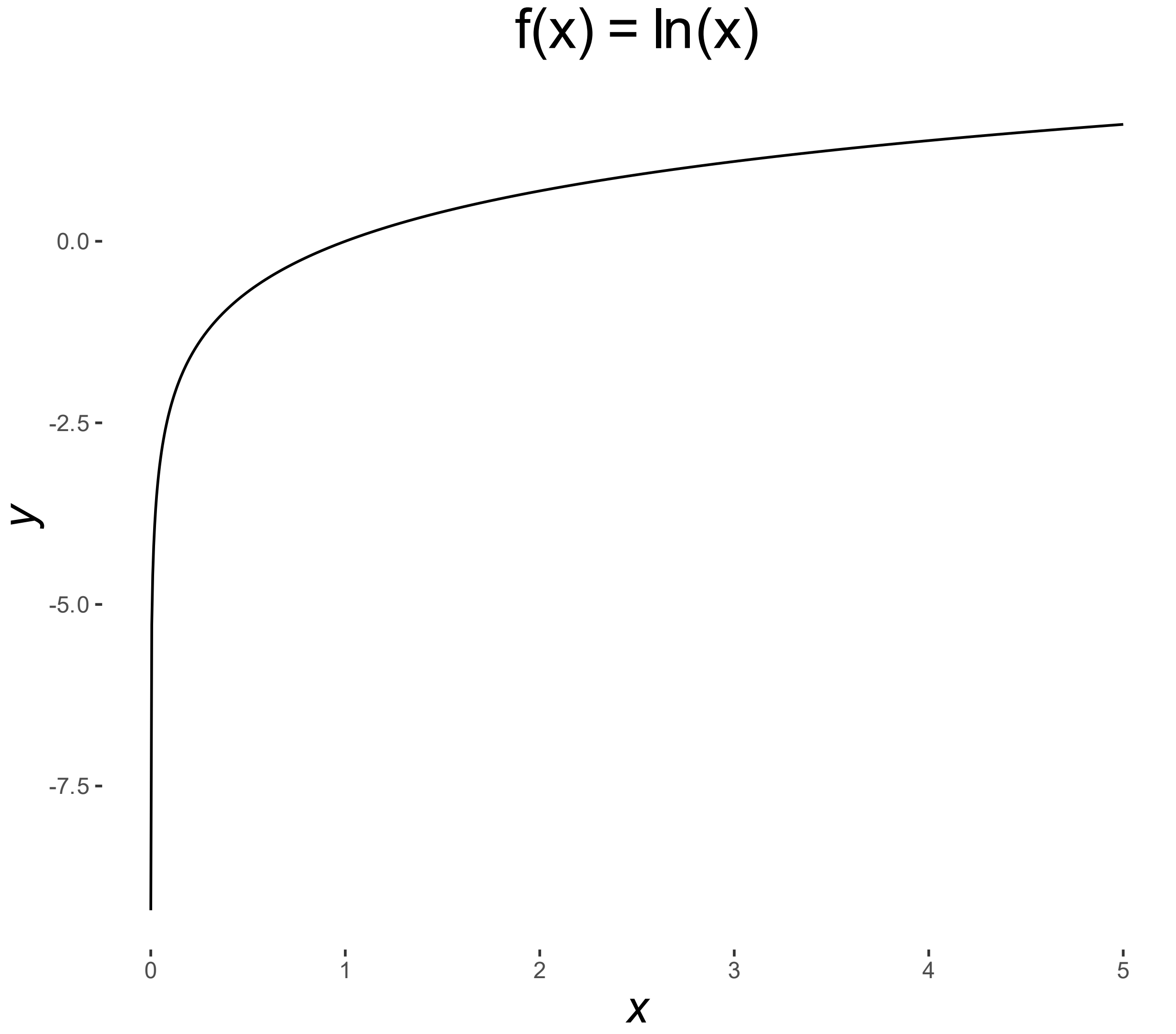
- When $x > 0$ the function is *monotonic increasing*.
- When $x < 0$ the function is *monotonic decreasing* and *asymptotic*.
- Any constant raised to the power of zero equals 1: $x^0 = 1$



Logarithmic functions

Logarithmic functions are the inverse of exponential functions.

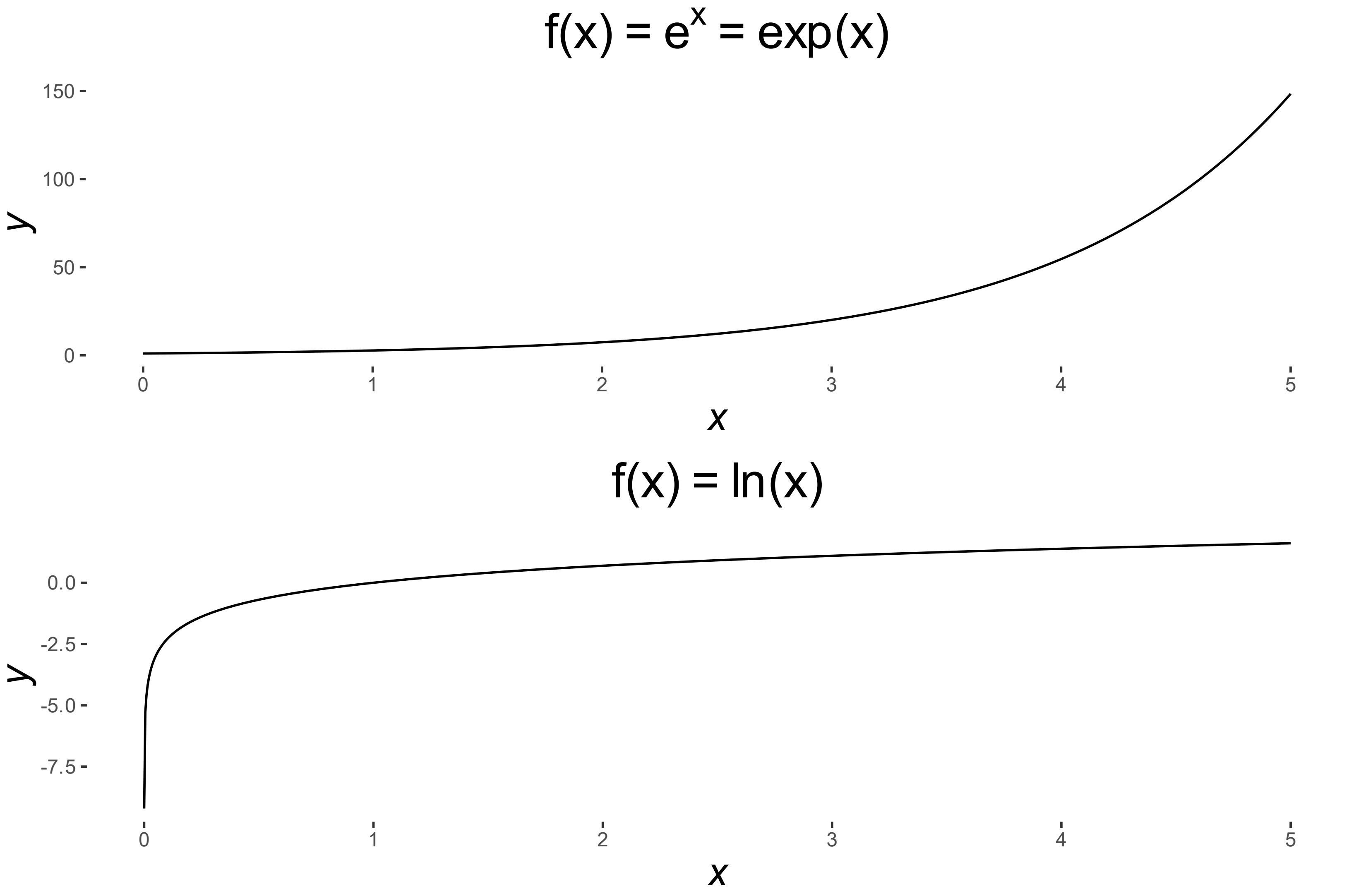
- Applying a logarithmic function *undoes* an exponential function.
- Logarithmic functions are slow-growing, but *not asymptotic*.



Exponential and Logarithmic functions

Mechanistic Interpretations

- Exp: *feedback* processes, exponential growth, divergent
- Log: *diminishing returns*, useful for dealing with very large number, *linearizing*, variance stabilizing

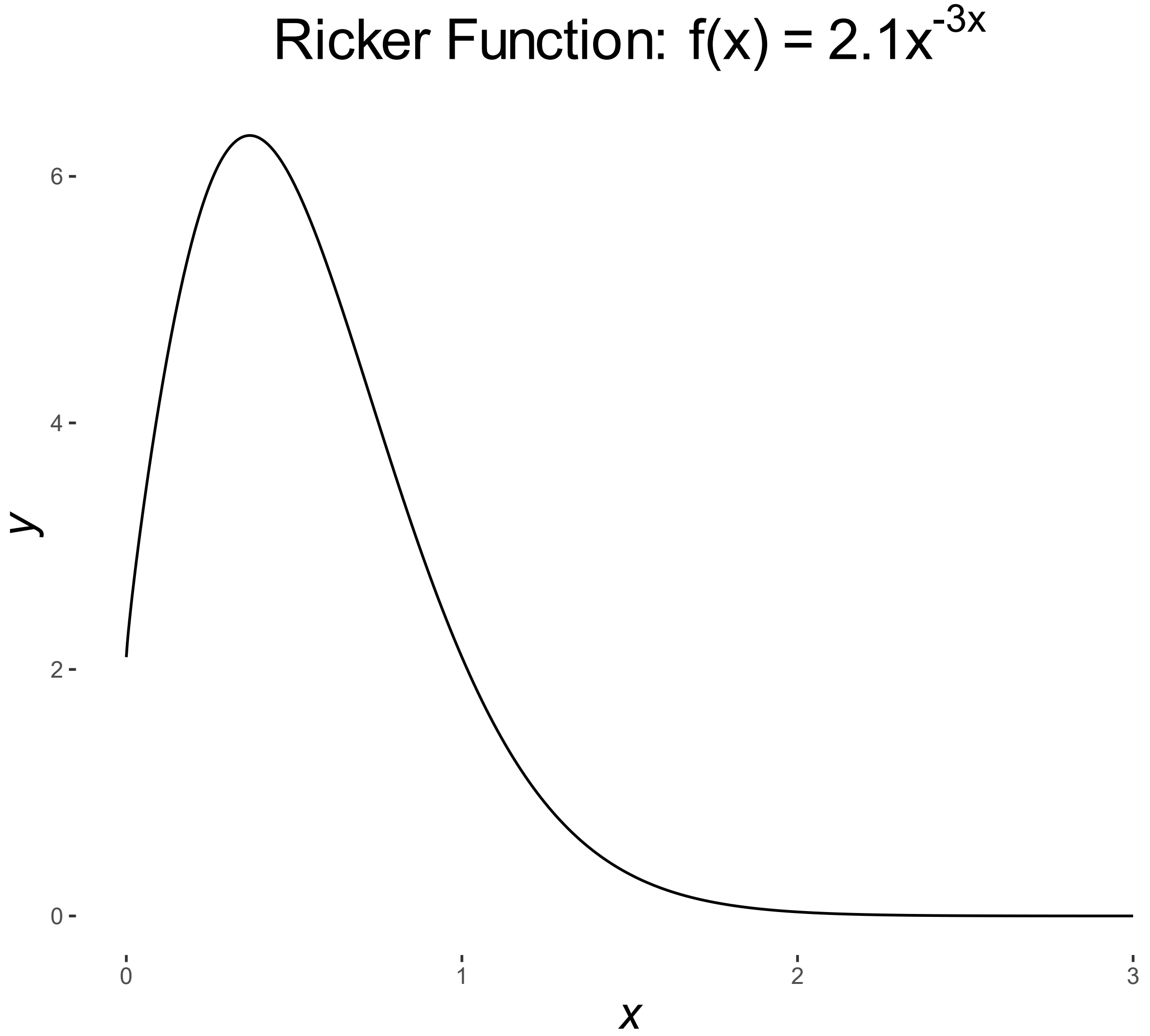


Hybrid functions

Just like the name says, they are mixtures of different function types.

- Often have a theoretical basis: they can be *mechanistic*.

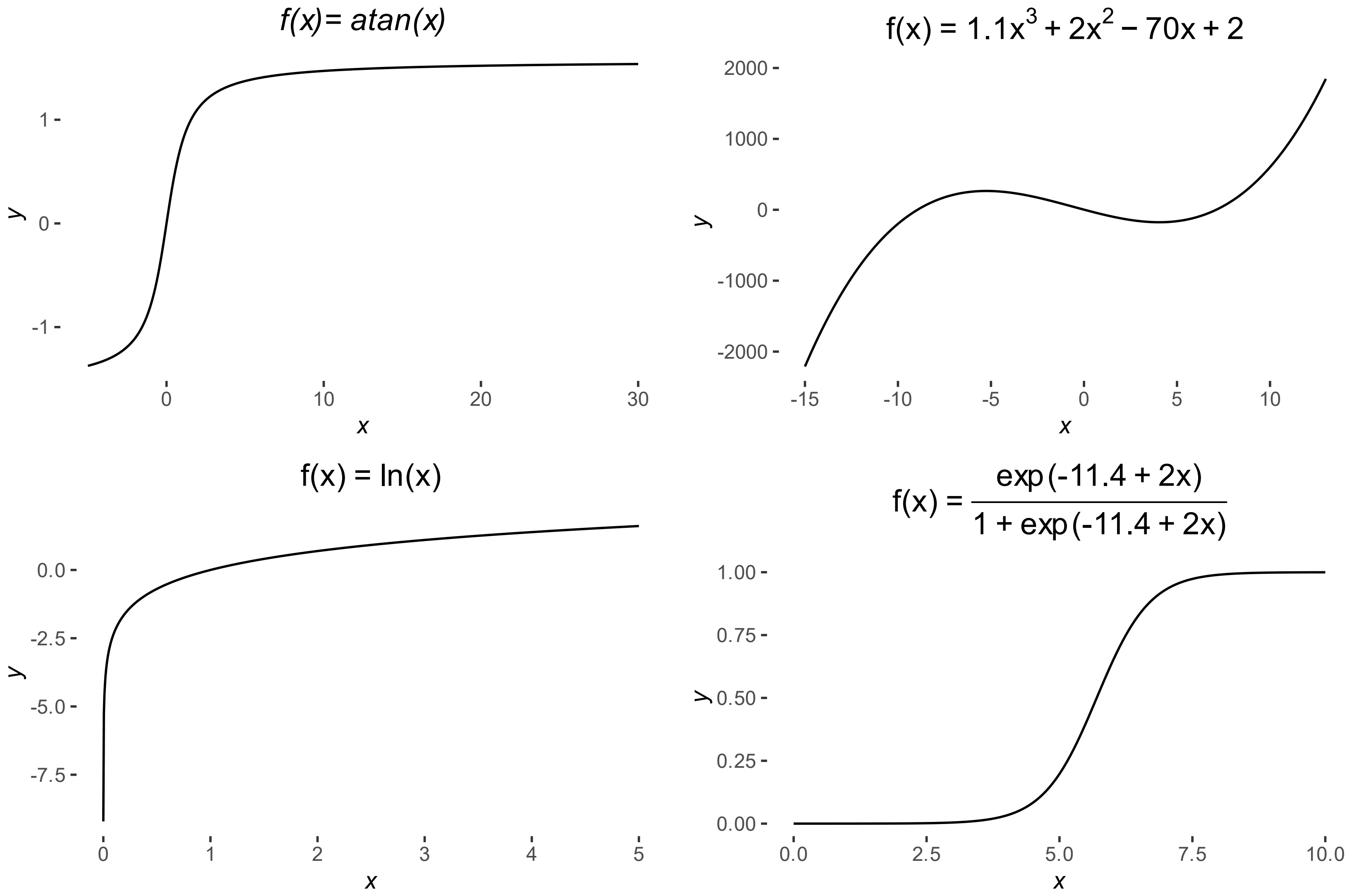
The Ricker function



Graphical intuition

Function Terminology

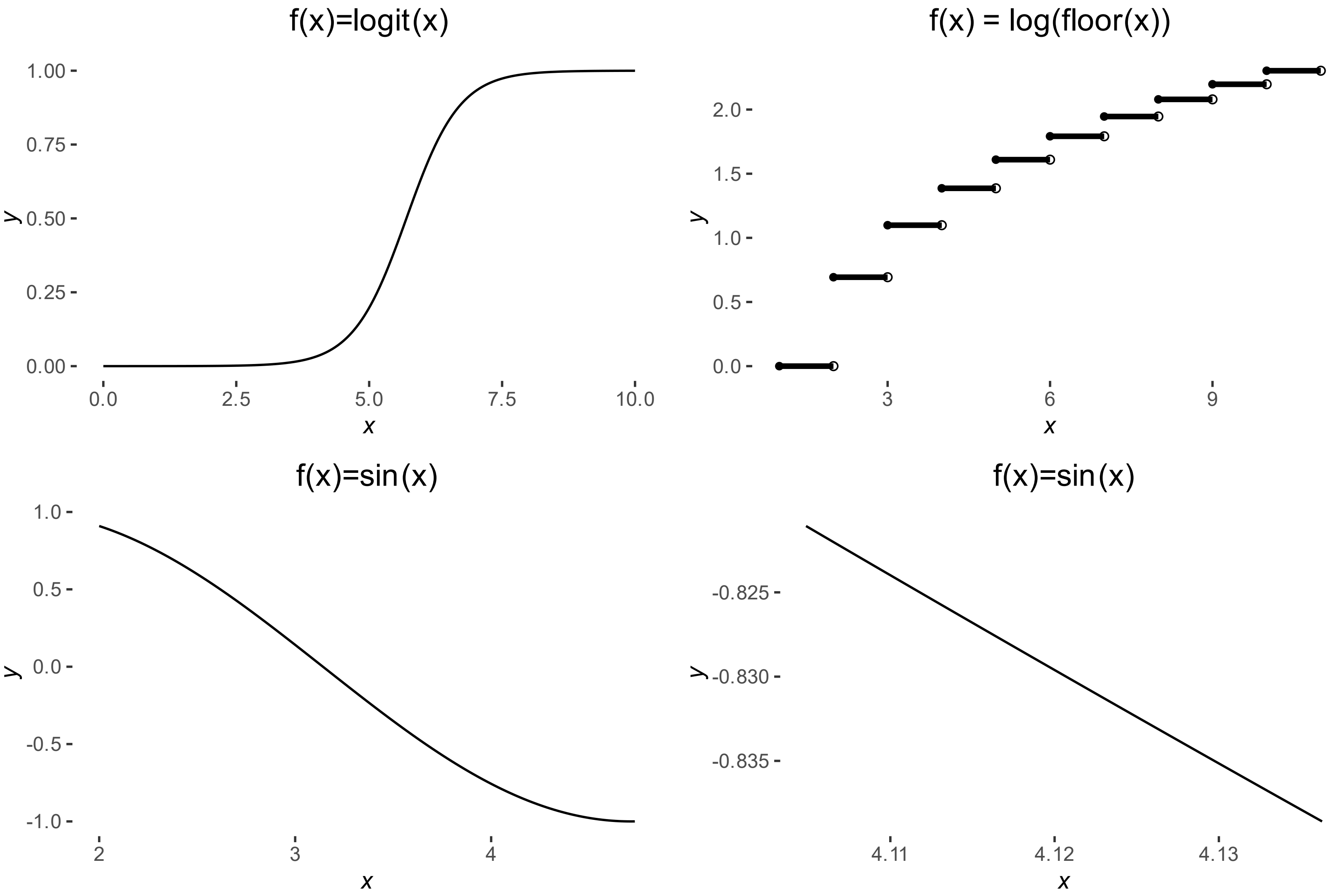
- Asymptotic: tends toward a value
- Divergent: tends toward infinity or negative infinity
- Monotonic: always increasing or always decreasing



Graphical intuition

Function Terminology

- Continuous: no breaks or jumps
- Local linearity: *most* functions resemble linear functions if you zoom in close enough.
  - This is closely related to **differentiability** in calculus.



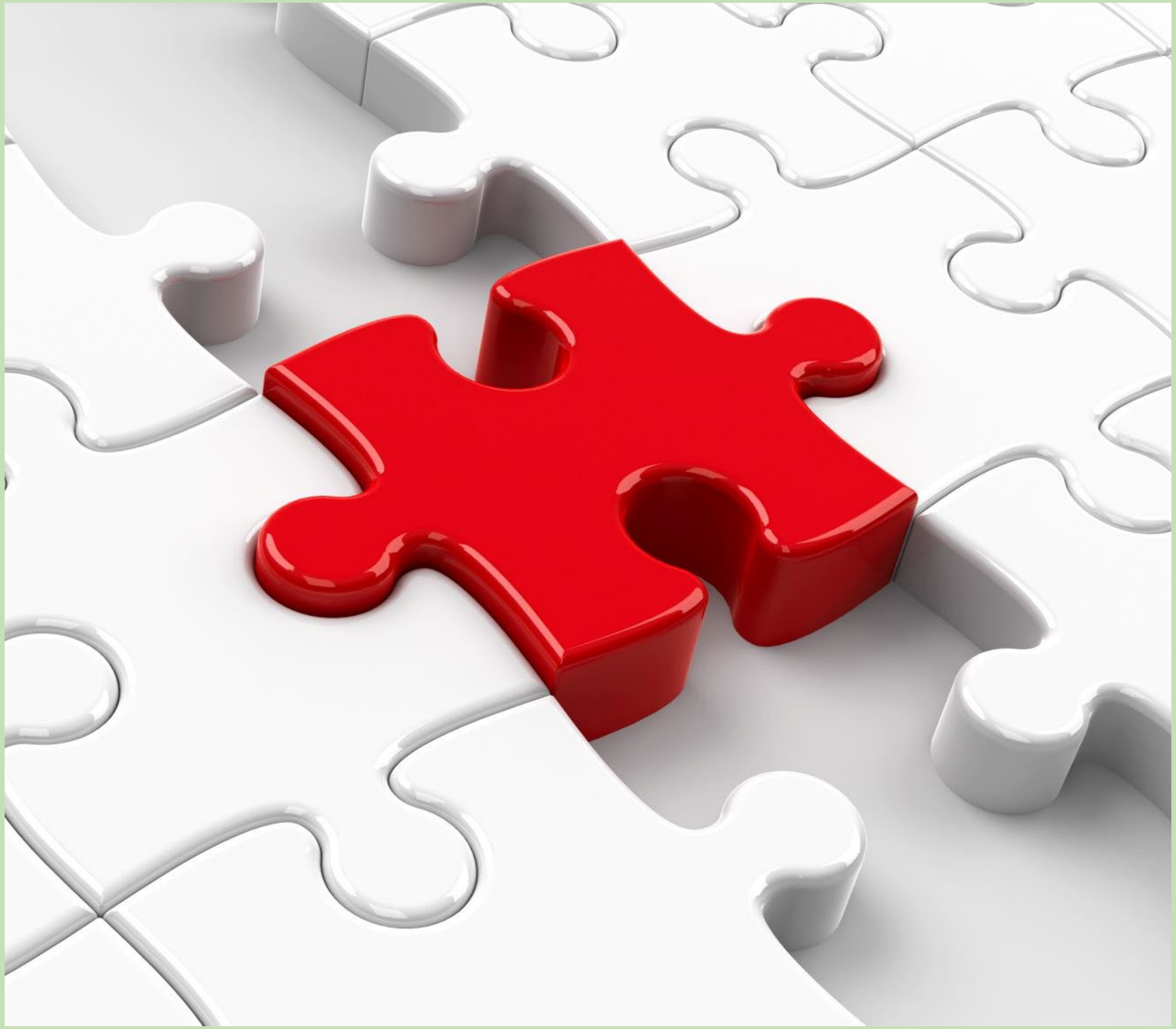
In-Class Reading Data Files

Probability Distributions 1

General Concepts

Key probability terms and concepts

- Inference with the dual model paradigm
- What is a distribution?
- Event, domain, sample space
- Key probability theory results
  - law of total probability
  - independent events



Stepping back: What do we need to do inference?

We need a model: a *dual* model!

Why do we want to do inference?

- We want to go beyond descriptive statistics.
- We want to learn something about a larger population from a sample.
- We want to estimate population parameters from sample statistics.
- We want to create a statistical model for understanding and/or prediction

Stepping back: What do we need to do inference?

We need the Dual Model Paradigm to do inferential statistics.

- We need the *deterministic* model of the means to about the *average* or *expected* behavior.
- We need the *stochastic* model to know about the variation.
- We need the *stochastic* model to know if an observation is *unusual*

Stepping back: What is inference?

For our purposes: inference is a way to learn something about a larger *population* from the properties of a *sample*.

More formally: Inference is estimating population *parameters* from sample *statistics*.

- We use the *deterministic* model to calculate model parameter estimates.
- We use the *stochastic* model to quantify *confidence* and *significance*.

Inference: why do we need distributions?

Couldn't we just use our deterministic model to make predictions?	Probability Distributions

• Sure, but without a stochastic model we can't quantify the uncertainty in our guesses.
• Relatively few systems are completely, or even mostly described by a deterministic model.
    • Planetary orbits
    • Chaotic systems governed by deterministic functions: sadly, we won't get to talk about these.
    • Logistic population growth
    • Lorentz equation
	• Help us understand the 'noise' part of the system.
	• Help us quantify and understand uncertainty.
	• Theoretical Distributions
	    • There are hundreds of named, parametric distributions
	    • Defined by mathematical functions
	    • Describe Stochastic processes
	• Empirical Distributions
	    • Calculated from data

What is a distribution?

Remember that words often have specific meanings in statistics:

- What do I mean by *likelihood*?
- What do I mean by *event*?

A distribution is a map from events to measures of likelihood

- Why would we want such a map?
- What do I mean by likelihood?
    - We'll talk about probability theory later.

Parametric and Empirical Distributions

Parametric distributions are defined by mathematical *functions*	**Empirical distributions are computed from *observations*.**
• The functions have one or more *parameters* that define how probabilities are allocated to events. • What are the parameters of the Normal distribution? • We often want to estimate the parameters from samples.	• There is no analytical function, but we can compare empirical distributions to parametric distributions. • Useful for comparing null and alternative hypotheses

Probability Distribution Functions

The map of events to probabilities are defined by:

- **Probability Density Functions** for continuous distributions
- **Probability Mass Functions** for discrete distributions.
- The values of PDFs and PMFs are always non-negative, by the definition of probability.

Two other types of functions are used to describe distributions:

- **cumulative functions**
- **quantile functions.**

Density or Mass Function: PDFs & PMFs

Probability density is the y-value of the probability density curve for a given value of x.

- You can think of it as the height of a curve
- For *continuous* distributions, it is *not* equal to the probability of observing a particular value of x.

Cumulative Probability Functions: CDFs & CMFs

Probability Density is the **height of the density curve.**

- Provides a measure of likelihood of an event
- Measure is relative for continuous; measure is the probability for discrete.

Cumulative density is the **accumulated area under the density curve** to the left of x.

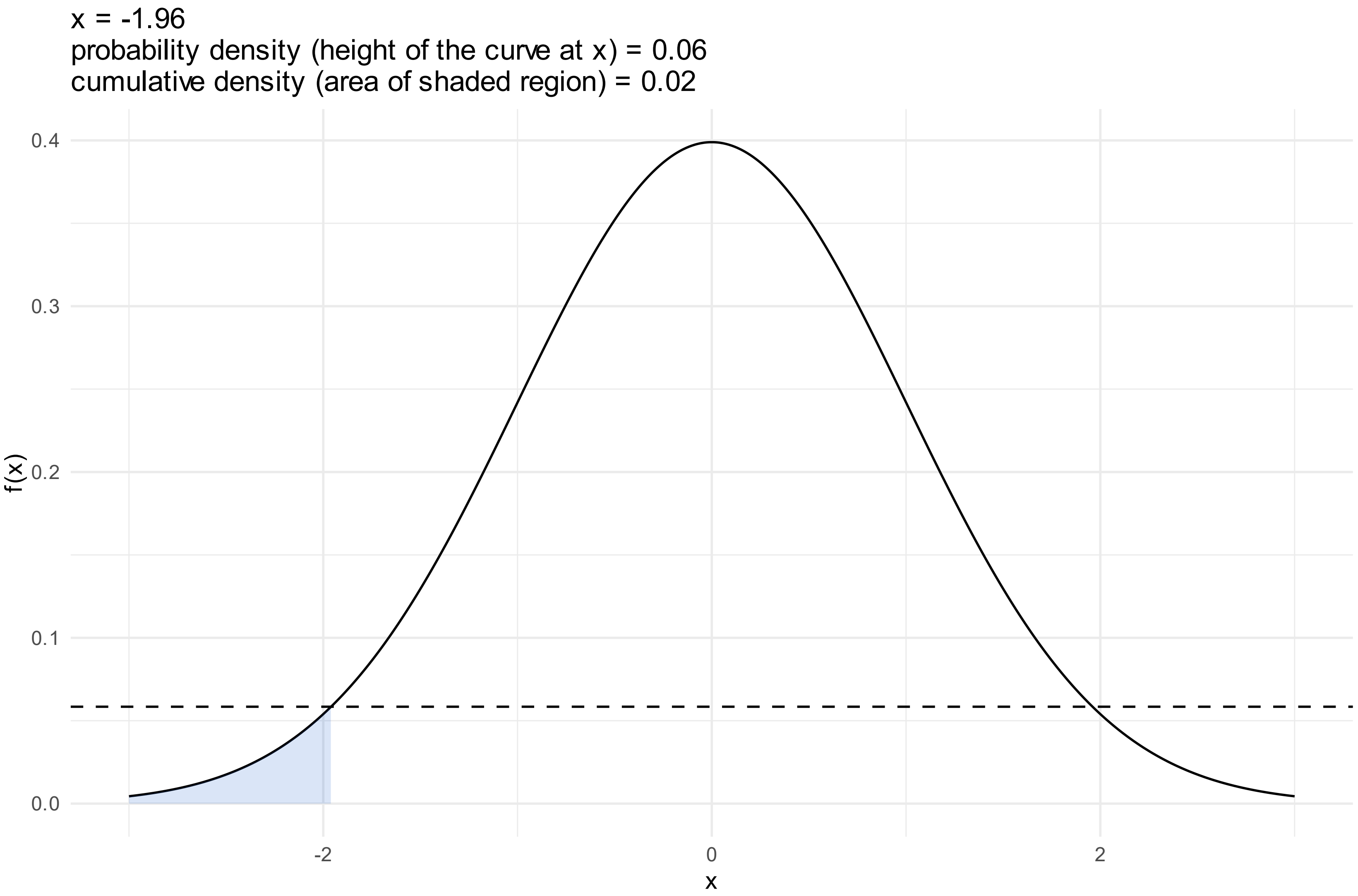
- It's an integral!
- It is the probability of observing a value equal to or less than x.

Probability Distribution Functions: Graphical Intuition

Demonstration of PDF and CDF using the Normal distribution.

Remember:

- *Density* = height of the curve at x.
- *Cumulative Density* = area under the curve, to the left of x



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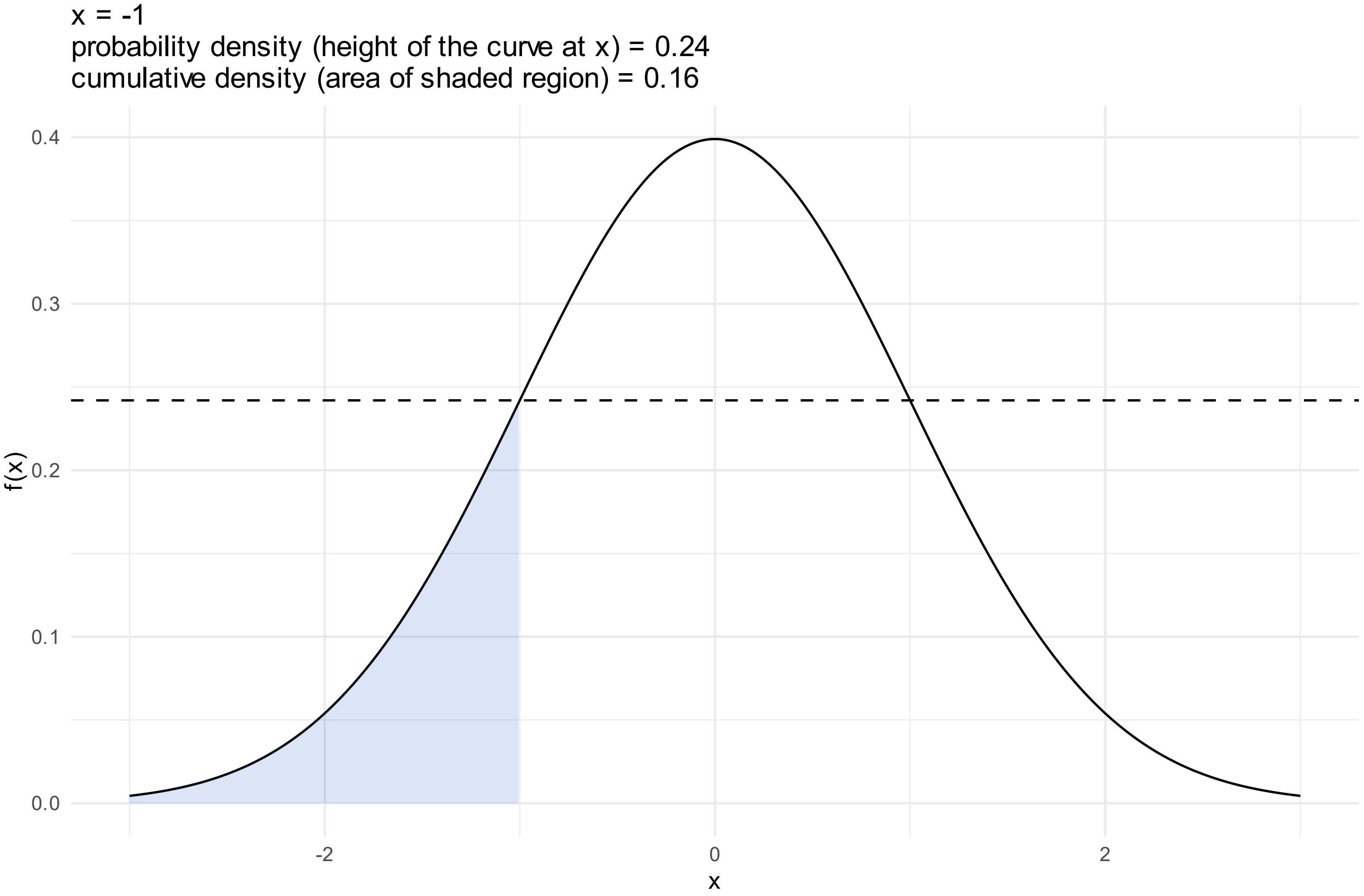

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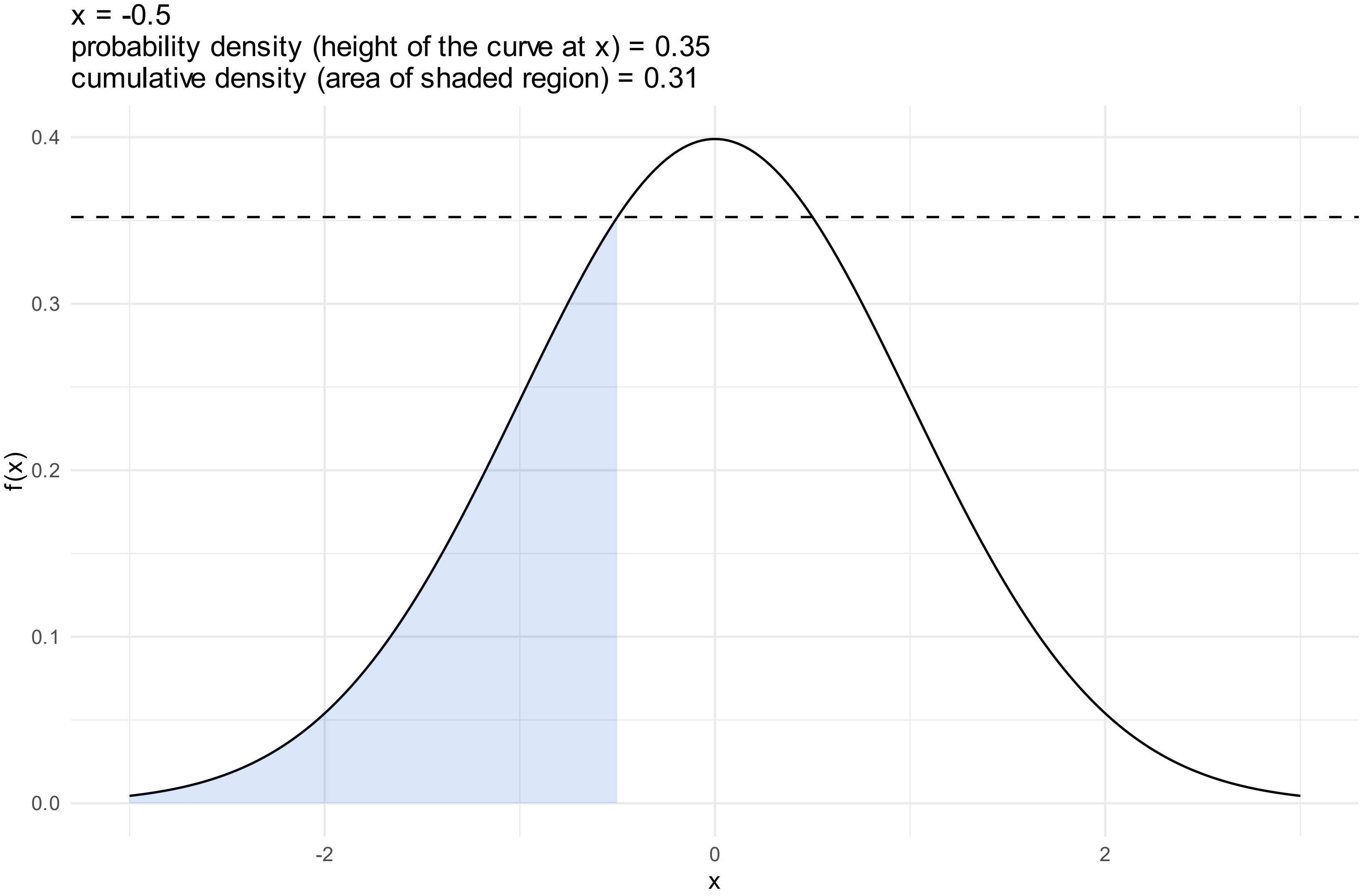

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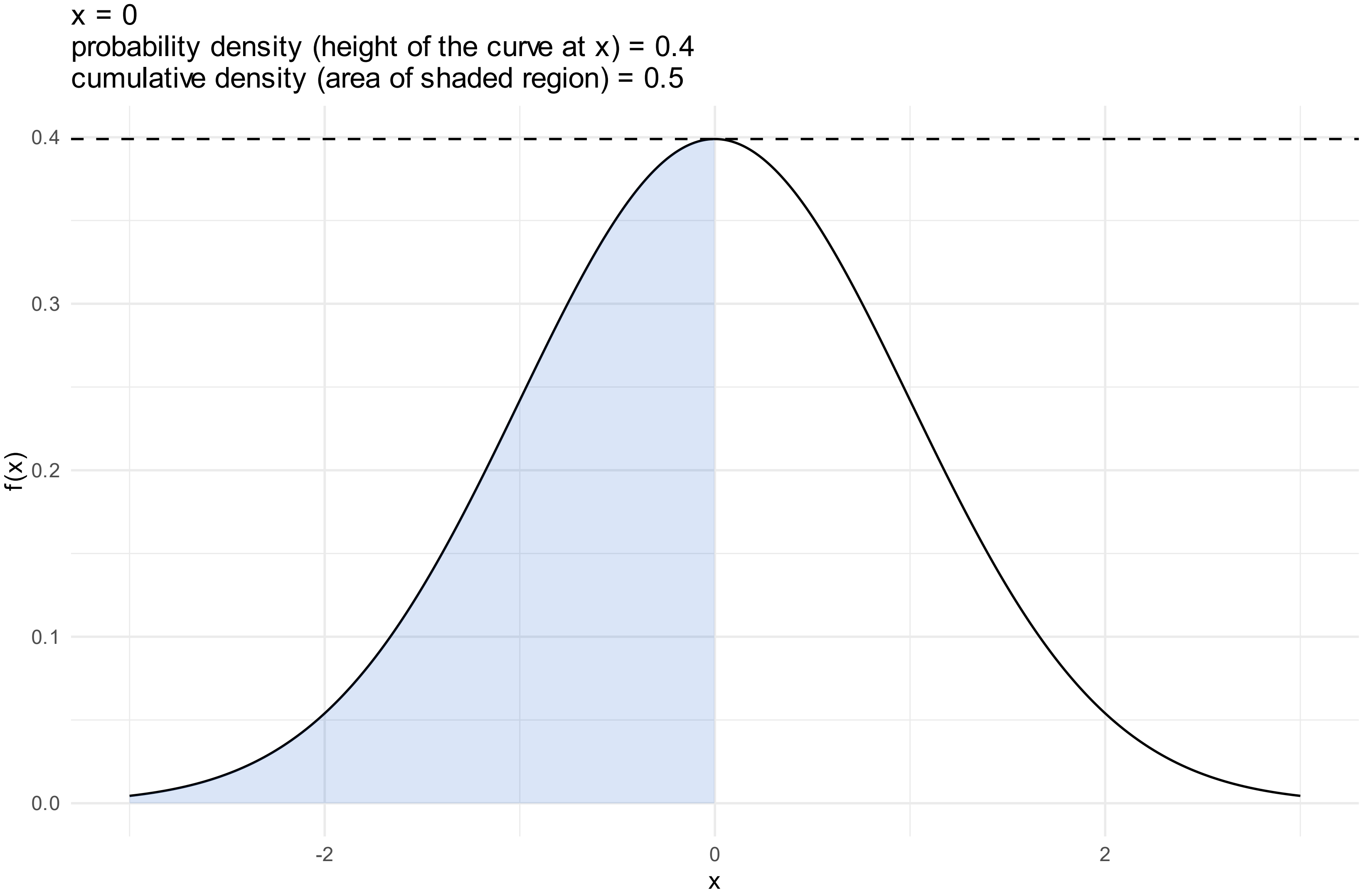

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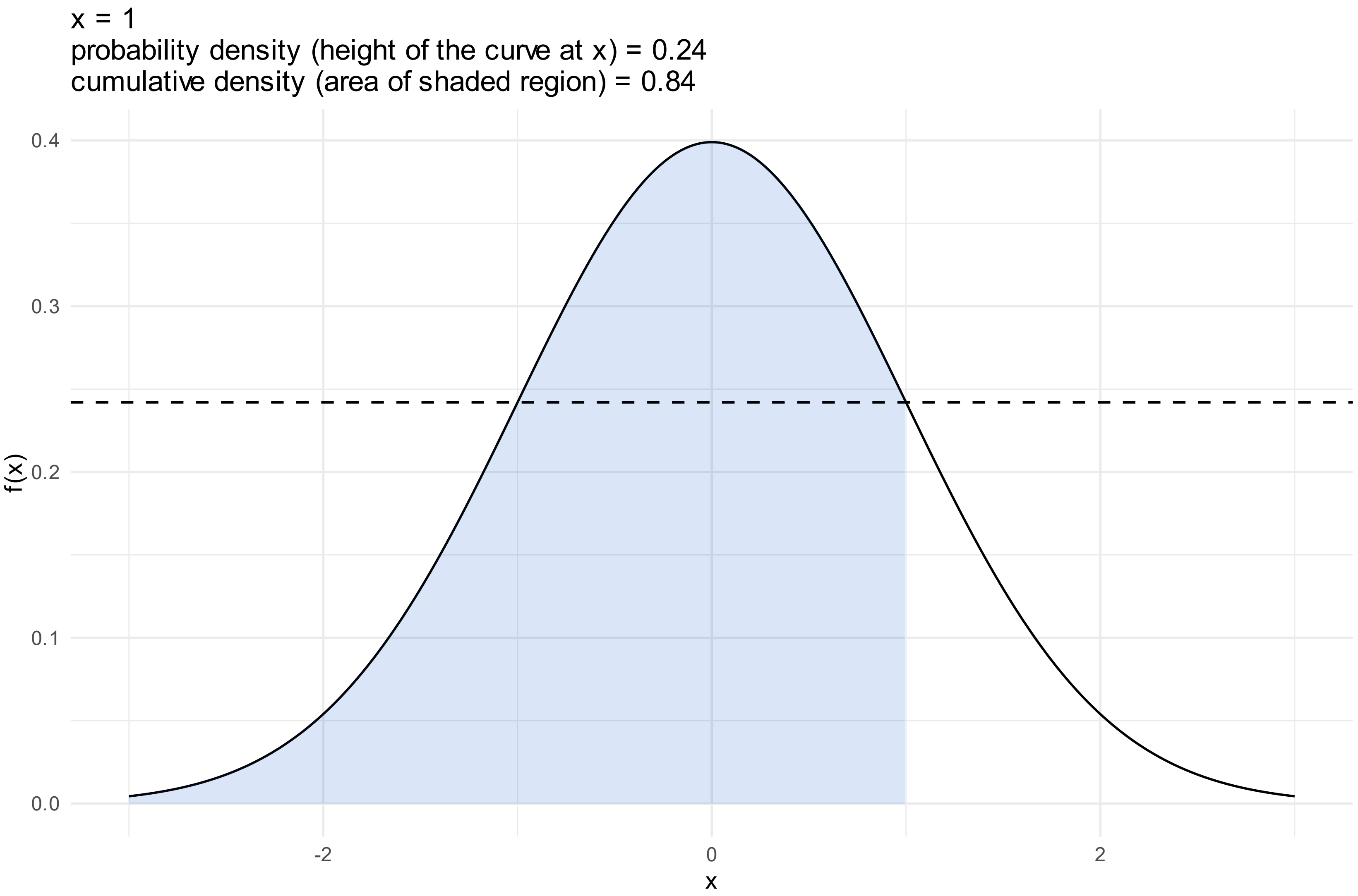

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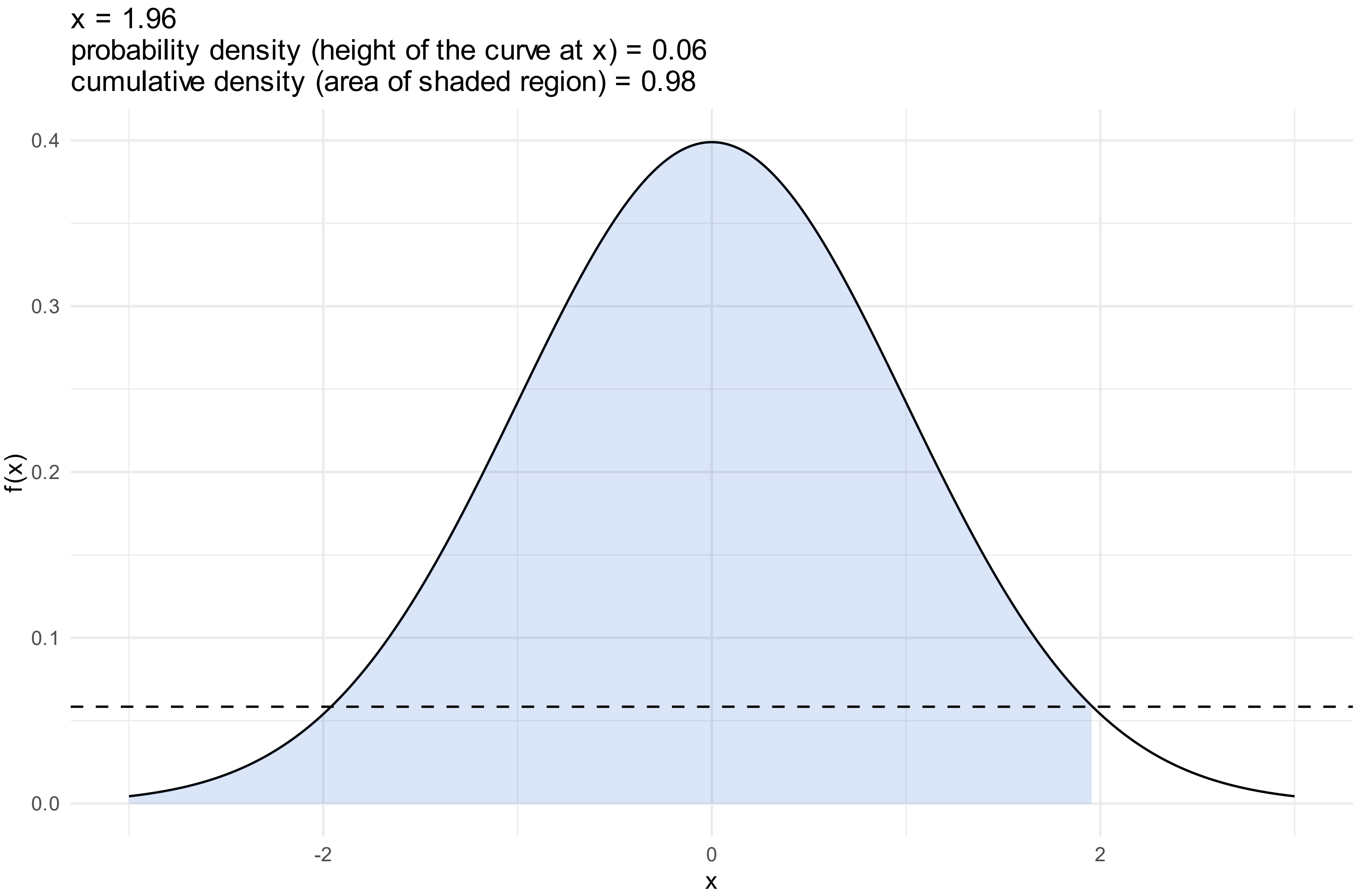

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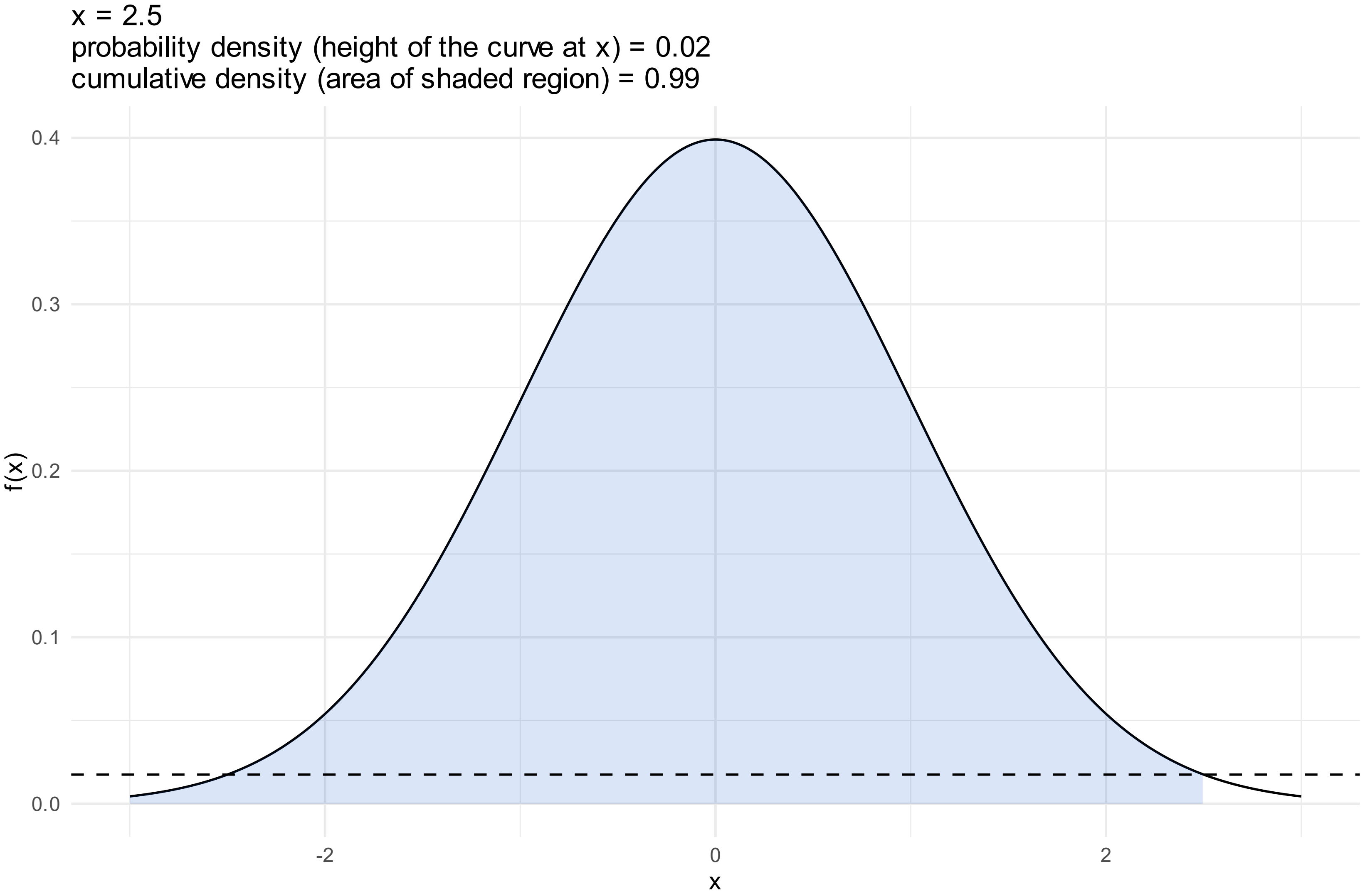

Recap of essentials:

Distributions

1. They assign a *probability* to every *event* in a *sample space*.
2. We can use them as the *stochastic model* in the dual model paradigm.

Probability essentials

1. Probabilities are non-negative
2. Law of Total Probability: Probabilities of all events in sample space sum to 1.0
3. Independent events: joint probability is product of individual probabilities

We'll continue to build our intuition about Probability Distributions